Volume XIX, No. 1

TEXAS MATHEMATICS TEACHER

January 1972

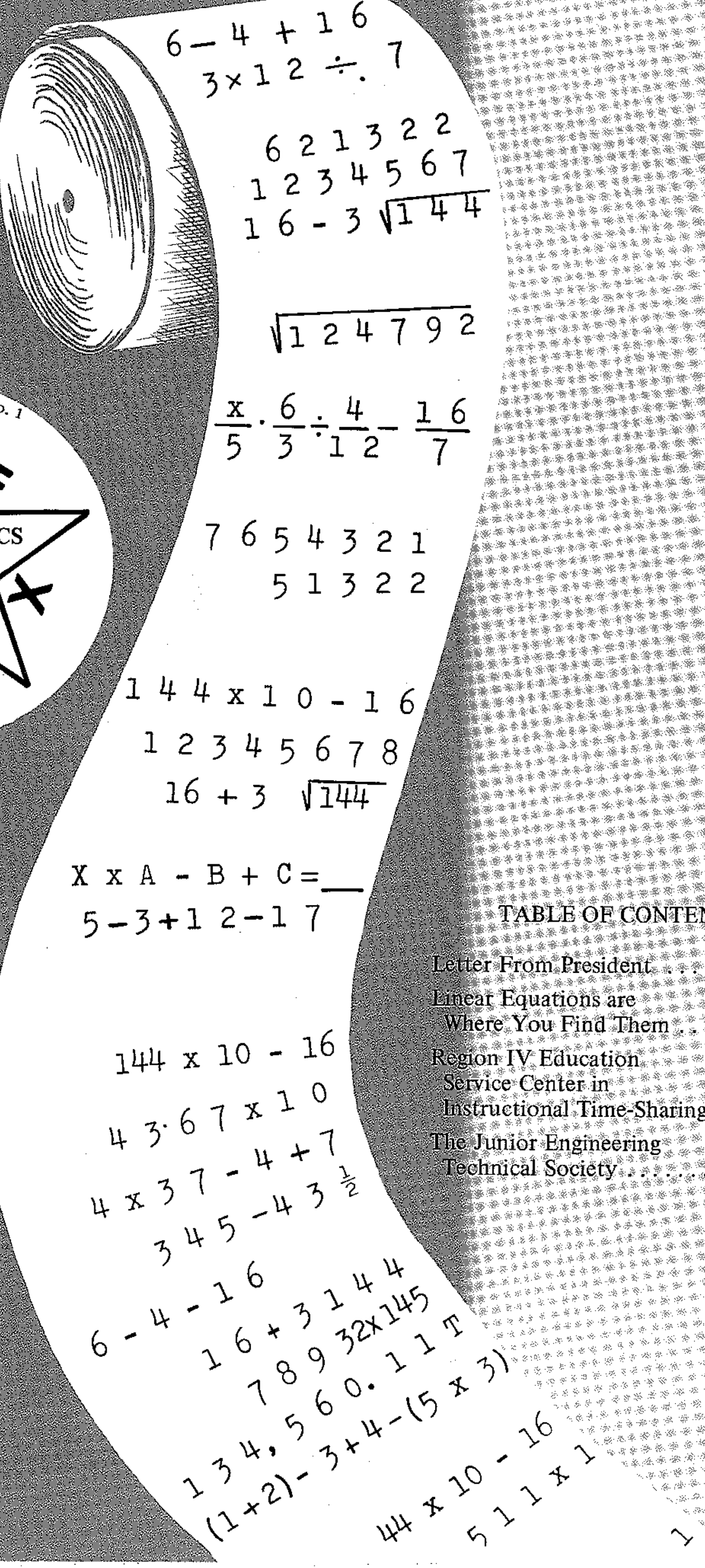


TABLE OF CONTENTS

	Page
Letter From President	3
Linear Equations are Where You Find Them	4
Region IV Education Service Center in Instructional Time-Sharing	8
The Junior Engineering Technical Society	10

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $2.00 entitles members to all regular Council Publications.

LETTER FROM THE PRESIDENT

This journal will be our third one and we have received many compliments on it. Lots of time and effort goes into the journal and we hope you will like it. Do you think that we should continue after you have seen three issues. Let us hear from you.

Again, I am making a plea for articles. This seems to be our biggest trouble. This is your chance to get an article published. Why not take time to write an article about something interesting that you are doing in your school. We are always glad to hear from all sections of the State. This is what the journal is doing. We like to keep posted on the things that are happening throughout the entire state of Texas. We have approximately three-hundred copies going to people outside the State and this is our chance to inform them as to what we are doing in Texas.

By the time this journal reaches you we will have had our C.A.S.M.T. Convention which we trust will be a success in our new convention location. There is some talk about the University of Texas giving it up on account of a lack of facilities. I would appreciate letters informing me as to whether the Texas Council of Teachers of Mathematics should take it over. We might even join together and have it jointly sponsored by T.A.S.M. and T.C.T.M. Let us hear from you.

We had a very successful workshop at J. Frank Dobie High School. Everyone seemed to be impressed. I wish to extend my thanks to the people who conducted the workshop; Mr. Bill Ashworth, and the mathematics department for sponsoring it; and Mr. Allen Sory, Principal, for letting us use his building free of charge. Everyone did a fine job and we thank you. We are hoping to have at least two more in the State before school is out.

I had a nice visit at a Saturday meeting in Nacogdoches with the East Texas Area Council of Teachers of Mathematics. I feel that everyone enjoyed the meeting. They are doing a fine job. Keep up the good work. Let us hear from some of our area councils throughout the State.

I think that you will be happy to know that T.C.T.M. membership has grown to well over one thousand. Thanks for all of the work that has been done throughout the State. You know the membership fee is the only source of revenue of the Council. We still have trouble finding money to accomplish the things that we would like to do. To all of our advertisers, may we say thanks for contributing and making the journal possible. Without you it would be impossible.

Have a nice holiday season, and we expect to see many of you in Beaumont in February. The program looks like they are going to have a great convention. The people responsible for putting the convention together are to be commended. ■

Coming Events:

NCTM Meeting in Beaumont, Texas, February 10-12, 1972. Good in-service-training opportunity for mathematics teachers.

April Gold in Chicago! or **Largest NCTM Meeting ever!** Date: April 16-19, 1972.

LINEAR EQUATIONS ARE WHERE YOU FIND THEM

DR. ARTHUR BERNHARDT
UNIVERSITY OF OKLAHOMA

1. REMEMBER THE AMOEBA.

A linear equation in one unknown has the pattern $ax = b$. It states that **b** is the product of the given number a with an unknown number **x**. The problem is to identify **x**. For example,

$$3x = 15 \qquad (1)$$

has the solution 'x is 5'; and this solution is unique. We may write

$$x = \frac{15}{3} \qquad (2)$$

Within fortran print format statements such a slash would indicate that the following 3 begins a new line, but here the slash indicates that the 3 should be written at the beginning of the **same** line. Thus, statement (2) **implies** statement (1), much as the **passive** sentence 'Abel was killed by Cain' implies the **active** assertion 'Cain killed Abel'. However, since the solution of equation (1) is unique we can consider the expression 15/3 as another name of the unique root x = 5.

I first met linear equations of this type in grade school. We constructed a multiplication table

1	2	3	4	5	6	...	
2	4	6	8	10	12	...	
3	6	9	12	15	18	...	
4	8	12	16	20	24	...	as far as 12 x 12.

The first row was a list of natural numbers from 1 to 12. The second row was the 'table of two's', beginning with 2 and increasing each time by 2. We called these numbers **even**, observing that odd numbers such as 1, 3, 5, ..., did not occur among the **multiples** of 2. Later I learned that numbers conspicuously absent from the interior are called **prime**; and those on the diagonal are **perfect squares.**

Multiplication tables are used for multiplying. To find '3 times 4' we looked in row 3 and column 4, finding the product 12. We knew that this was also '4 times 3' since in every instance the **k**th row was the same entries as the kth column, a property the new mathematics calls the **commutative law.**

We learned to use the multiplication table for **division** also. To find 36 ÷ 4 we scanned row 4 until we found the entry 36, which occurs in column 9. This is the quotient, 36/4.

In other words, we discovered an **algorithm** for solving linear equations: to find the root of $ax = b$ look in row a until you locate the entry **b**. The label at the top of this column is our answer.

The foregoing **division algorithm** has its limitations. Given the equation 3x = 13 we scan row 3 but never find 13. However, we find 12 in column 4 and 15 in column 5. Since the table entries bracket the dividend 13, we might conclude that the root x lies **between** 4 and 5. But if we stick to natural numbers we must confess that 13 ÷ 3 does not lead to a natural result. The algorithm aborts although 12 ÷ 3 would have been exactly 4. We say that 13 ÷ 3 has the quotient 4 and the remainder 1. This modified algorithm is also called **division** for it includes the **inverse** of multiplication as a special case, whenever the remainder is zero.

These considerations show that, within the set of natural numbers, $ax = b$ does not always have solutions. Otherwise states, $x = b/a$ is not necessarily the **name** of a unique number.

Enlarging our domain to include the entire **field** of non-zero rational numbers restores the comfortable property of **closure.** In a rational field **each** linear equation $ax = b$ has the unique root $x = b/a$. Is this intimate connection between linear equations and the rational number system an insight of the new mathematics? As Merle Mitchell commented Thursday on the Rhind Papyrus, the ancient Egyptians had a special character for the rational root of 3x = 2.

2. IT'S A MOD WORLD.

Let us construct a mod **p** multiplication table, say for p = 5.

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

The table was originally 5 x 5 but I took the liberty of omitting the fifth row and column, since 5 = 0 times any number is always again 0.

Again we locate the product 'ma' in row **m**, column **a**. The entries in each row are **distinct** since ma = mb would imply a = b, and the table was constructed to make all first row entries distinct. This argument was valid for the multiplication table for natural numbers. But in a finite mod **p** field 0 cannot occur as a product of non-zero factors, so **each** row is a **permutation** of the first row. The same is true for each column, so our 4 x 4 table is a Latin square.

It is easy to see that the linear equation ax = b has a unique root in the mod **p** system, without having to extend the field to include non-integral factors. We solve 3x = 2 by scanning row 3. We find the numerator 2 in column 4 so that the **numeral** 2/3 reduces to the integer 4. It is readily verified that x = 4 is a root, since 3x = 12 = 2 mod 5.

For any mod **m** system where **m** is not prime, its multiplication table is no longer a Latin square. Correspondingly linear equations do not necessarily have unique solutions.

Thus, 3x = 2 has no solution **mod 6** while 2x = 4 has two roots $\{2, 5\}$.

3. LABELS ARE OFTEN LIBELS.

Returning to the field of real numbers consider now a linear equation in two unknowns

$$3x + 4y = 60 \tag{3}$$

Its solution is the set S of all ordered pairs (x, y) which satisfy this relation. Thus, (20, 0) **belongs** to S but (5, 6) **does not.**

A set may be defined **by rule** or **by roster.** But equation (3) has too many solutions to be listed in a roster. For, if we substitute x = a into (3) it reduces to

$$4y = 60 - 3a \tag{4}$$

which is a linear equation in the one unknown **y**. Let **b** represent the unique root of (4). The (a, b) is a solution of (3). Accordingly there is an ordered pair (a, b) satisfying (3) for each real number x = a.

We define the solution S of equation (3) by the **rule**

$$S = \{(x, y):\ 3x + 4y = 60\}$$

In plane analytic geometry (x, y) is the name of an arbitrary point in that plane. Consequently, S is a **point-set** (or **locus**), namely the set of all points whose coordinates satisfy the **rule** specified by equation (3).

It can be shown that S is a **straight line.** The corresponding **rule** is for this reason called **linear.** The reader is warned that in 3-dimensional space the set $T = (x, y, z):\ 3x + 4y + 5z = 60$ is a **plane.** Yet nobody calls this equation 'planar' but rather **linear** equation in three unknowns. Etymology are interesting, but are often overruled by usage.

In algebra today **linear** means **first degree.** The equation $ax^2 + bx + c = 0$ is linear if a = 0 and $b \neq 0$. The equation ax + by = r is **not linear** if both **a** and **b** are zero, but is linear otherwise. Incidentally, an equation of **second** degree is called quadratic, from the Latin word **quator** for 4. The exponent 2 is similarly misnamed.

Consider two linear equations

$$\left.\begin{aligned} ax + by &= r \\ cx + dy &= s \end{aligned}\right\} \tag{5}$$

with solution sets R and S, respectively. We shall examine the union and intersection of these sets.

The set union R S of these loci has the defining rule

$$(ax + by - r)(cx + dy - s) = 0 \tag{6}$$

On multiplying this out we obtain a quadratic equation in two unknowns

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \tag{7}$$

What is the condition on the six coefficients of an arbitrary quadratic form to guarantee that it will factor? It is the vanishing of the determinant

$$T = \begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix}$$

When T = 0 the graph is called **composite** by optimists, and **degenerate** by pessimists.

It would be premature to say that T = 0 always corresponds to a composite of **two** straight lines. Whenever the coefficients. are proportional, a:c = b:d = r:s, the two lines coincide.

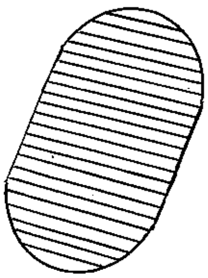
Moreover, $x^2 + y^2 = 0$ is factorable only with the help of complex numbers: (x + iy) (x - iy) = 0.

There is only one real point (x, y) on each of these 'imaginary' lines. We spot these potential trouble makers with the help of the discriminant: $\triangle = B^2 - 4AC$. In case T = 0 and $\triangle$ is positive, the composite graph (7) is two intersecting lines. In case T = 0 but $\triangle$ is negative, we have a pair of imaginary lines, sometimes called a **point ellipac.** In case both T and $\triangle$ vanish we have parallel or coincident lines. For $x^2 - 2xy + y^2 + 1 = 0$ the parallel lines are also imaginary!!

The array T suggests three linear equations

$$Ax + By + D = 0 \qquad (8)$$
$$Bx + Cy + E = 0 \qquad (9)$$
$$Dx + Ey + F = 0 \qquad (10)$$

Do these three linear arrays have any connection with the locus of (7)? By the way, let's call it a **conic** (a hyperbola, if $\triangle$ is positive; a parabola if $\triangle$ is zero; and ellipse if $\triangle$ is negative.)



The midpoints of horizontal chords lie on the line (8). The midpoints of vertical chords, you guessed it, lie on line (9). The intersection of (7) and (8) bisects every chord, and gets our vote for the **center** of the conic. What about line (10)? If the conic passes through the origin, it is the tangent to the conic at the origin.

4. THE ROAD TO MATRICES IS PAVED WITH LINEAR EQUATIONS.

Instead of their union consider the set **intersection** of two lines. It consists only of those points (x, y) which satisfy **both** rules. We rewrite these simultaneous conditions:

$$\begin{array}{c} \boxed{x \quad y} \\ \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array} = \begin{bmatrix} r \\ s \end{bmatrix} \qquad (11)$$

Equation (8) may be regarded as a single **matrix** equation

$$a^* x^* = r^*$$

where asterisks lable the corresponding **arrays.** Given the coefficient matrix

$$a^* = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and the vector array $r^* = (r, s)$ we seek a point $x^* = (x, y)$ satisfying the linear (!) matrix equation (11). If the two lines coincide there are infinitely many solutions. If the lines are parallel the solution set is empty. We guarantee closure by outlawing both of these cases, demanding

$$ad - bc \neq 0$$

Sometimes we like to say that multiplication by a^* **transforms** the point (x, y) into the point (r, s).

In Newton's famous equation $m\vec{a} = \vec{F}$, multiplying the acceleration vector **a** by the mass **m** yields a force $\vec{F}$ in the same direction as $\vec{a}$. But, equation (11) shows how multiplying a vector x^* by the matrix a^* can change its direction also.

5. RICH MEN ARE POOR MEN WITH MONEY – Will Rogers

In the past decade we have been urged to teach inequalities. But, as I see it,

$$a < b$$

is just an abbreviation for the equality

$$a + p = b$$

when **p** is an unspecified positive number. Replace $<$ by $+ p =$

Positive numbers are a distinguished subset of the reals. Given two positive numbers p_1 and p_2; their **sum** $p_1 + p_2$ and their product $p_1 p_2$ are also positive. What is distinctive about a number p? It is a **square** (but not zero). If $p = a^2$ and $q = b^2$, clearly $pq = (ab)^2$ – multiplication closure. An appeal to the Pythagorean Theorem proves additive closure, $a^2 + b^2 = c^2$.

6. DRUNK SNAKES AND VICIOUS CIRCLES.

Two equations $f = 0$ and $m = 0$ are said to be the parents of a family $f + am = 0$, where **a** ia an arbitrary constant. Thus, the parabolas $y^2 = 12x$ and $x^2 = -10y$ generate the circle

$$x^2 + y^2 = 12x - 10y \qquad (f)$$

If both parents are circles we usually expect a family of circles. But, imagine how shocked the parents (f) and (m) would be

$$x^2 + y^2 = 169 \qquad (m)$$

to find a black sheep

$$12x - 10y - 169 = 0 \qquad (13)$$

in their family. They would likely call it a **radical axis.**

Parent m passes through (5, 12) as can be verified by substitution. the **half** substitution $xx = 5x$ and $yy = 12y$ yields a linear equation

$$5x + 12y = 169 \qquad (t)$$

For the general conic (7) let matrix T^* transform (a, b, 1) into (L, M, N). Then, a half-substitution gives its tangent line

$$Lx + My + N = 0$$

at the point.

Let $f(x, y) = 0$ be a parent equation, and let $f^* = f(a^*, b^*)$ for the substitution $x = a^*$, $y = b^*$. If $f^* = 0$, clearly the point (a^*, b^*) lies on its graph, F.

If f^* is **not** zero (a, b) is **not** on the graph F. There are two ways for a real number not to be zero. If F is a line there are two sides (i.e., half planes) which are off the line. It is not hard to guess that f^* positive corresponds to one side, f^* negative refers to points on the other side.

For two parents f and **m** the third equation

$$\begin{vmatrix} f & 0 & f^* \\ f & 0 & g^* \end{vmatrix} = 0$$

is a locus which passes through the point (a, b) and **through each point** common to both parents!!

As a particular instance

$$\begin{vmatrix} x & = & a & = & 0 & x^* & - & a \\ y & = & b & = & 0 & y^* & - & b \end{vmatrix}$$

is the **line** determined by the two points (a, b) and (a^*, b^*).

"FREE, EVERY MONDAY THROUGH FRIDAY **Mathematical Knowledge.** BRING YOUR OWN CONTAINER."

REGION IV EDUCATION SERVICE CENTER IN INSTRUCTIONAL TIME-SHARING

JAY JUSTICE

There are 20 regional Education Service Centers geographically located throughout Texas and Region IV with administrative offices at 202 North Loop West in Houston, is the largest of the centers population-wise. Region IV serves a seven-county area containing 56 school districts having more than 600 campuses and an enrollment in excess of 500,000 youngsters.

The regional Service Centers came into being in 1967 by mandate of the 59th Legislature. Briefly, the Texas concept of a regional Service Center is "an educational institution established by the state to provide services for school districts more economically than they might provide for themselves. A regional Education Service Center does not exercise control over local school districts. Its function includes the providing of assistance in planning toward innovative programs in education. District participation is completely voluntary."

Regional Education Service Centers are non-profit organizations funded from local, state and federal sources.

In the development of Region IV's computer services, it was foreseen that districts would have computer accessories that would connect to its time-sharing computer. These accessories would allow different locations within many districts to have simultaneous direct communication with the computer. One of these accessories is the teletypewriter terminal for use as an aid in classroom instruction.

In the summer of 1970, Region IV offered teletypewriter terminals to its districts. The first year response was tremendous with 80 terminals in use in 18 public school districts, three private schools and one four-year college. During the school year 1970-71, the teletypewriter terminal was used on 86 campuses by 300 teachers and approximately 38,000 students. Each terminal connects to the computer by means of an ordinary desk-type telephone, allowing students and faculty to "talk" to the computer in the BASIC programming language.

BASIC, a pseudonym for Beginner's All-purpose Symbolic Instruction Code, was written by Drs. Kemeny and Kurtz of Dartmouth College. The language is powerful yet easy to learn by virtually any ability group in grades 4-12.

In utilization of the teletypewriter, emphasis is on problem-solving and integration of the terminal into the existing curriculum. Students learn to model mathematical concepts via the BASIC programming language. A solution of a mathematically based problem is devised in a logical sequence of steps (an algorithm), these steps are converted to the BASIC language which the computer understands, and the computer then checks the student's model. Computer checking of a student's model gives immediate reward for correct work or points out a student's errors causing student refinement of the model. The error detection role of the computer is beneficial in that it tends to remove the teacher as the adversary in the classroom.

An example of this process is the concept of solving a quadratic equation. With the development of the quadratic formula, the student is asked to write a BASIC computer program to find the solution set for:

$$ax^2 + bx + c = 0, \text{ given } a, b, \text{ and } c$$

No additional information or warnings are given. In checking the assigned values of a, b, and c through his model, the computer informs the student he has attempted to take the square root of a negative number. The student then realizes he must refine his model by checking the value of the discriminant. In a similar manner, he will also refine his model to handle the $a = 0$ situation.

Course Offerings

Districts offer some courses which deal only with the use of the terminal and the BASIC language. This course, entitled *Computer Mathematics,* discusses methods of doing previously known mathematics and new mathematics by means of the computer and the teletypewriter terminal. Courses are also offered in Computer Science which deal more specifically with the inner workings of the computer itself.

However, in order to reach a greater number of students, Region IV emphasizes the integration of the teletypewriter terminal into the entire mathematics curriculum; thus, very little Computer-Assisted-Instruction (CAI) work of the drill and practice type is being applied region-wide. As an example, Houston Independent School District has prepared, with Region IV's assistance, a two-week *Computer Appreciation* course which they introduced to more than 6,000 eighth grade students last school year. Also, in the school year 1971-72, four school districts (Galveston, Houston, Pasadena and Spring Branch) will offer 22 sections of second year algebra and trigonometry using the text *Second Course in Algebra and Trigonometry With Computer Programming,* a publication of the Colorado Schools Computing Science Curriculum Development Project. This text is for a year's course in second year algebra and trigonometry, integrating the mathematics with the computer terminal and the BASIC language as an instructional aid and modeling device.

Users Meeting

On Saturday morning, February 20, 1971, Region IV sponsored a teletypewriter Users Meeting at J. Frank Dobie High School, Pasadena. The 121 teachers and administrators in attendance shared their experiences in sectional meetings in the different fields of mathematics and science (at least one such Users Meeting is planned for the school year 1971-72).

In preparation for the February Users Meeting, Region IV compiled and published a bibliography of materials concerning the use of the computer in the classroom. A representation of the teletypewriter keyboard was also prepared for aiding teachers in classroom presentations.

Student Contest

In the late Spring of 1971, Region IV sponsored a student contest in the BASIC programming language. Even in the short length of time the students had to prepare their entries, many excellent ones were among the 40 entries from nine districts. Entries were submitted from students in grades 7-12 and concerned such programs as chess playing, checker playing, randomly generated poetry and assembly language simulation.

Region IV will sponsor the Second Annual BASIC Student Programming Contest in 1971-72 and plans call for students to have four months in the preparation of their projects.

(Continued on page 10)

In The Future

To date, Region IV has received requests for 120 teletypewriter terminals from 22 public school districts, four private schools and one four-year college for the coming school year. This 50% expansion is due to new districts entering the program and an increase from previous user districts wishing to expand their instructional program in this area.

THE JUNIOR ENGINEERING TECHNICAL SOCIETY (JETS)

*J. G. McGUIRE, P. E.

The Junior Engineering Technical Society (JETS) is a national society with chapters in many high schools throughout the United States. Since 1955 the State Headquarters for the Society in Texas has been at College Station, Texas. The program sponsored by the Texas State Headquarters for JETS is built around service to the schools. The purpose of JETS is to encourage scholarship in such areas as mathematics, science, and English, and to acquaint the student, his teachers, and his parents with the nature of engineering and what preparation the student needs to make in high school in order to be successful in college study. High Schools are encouraged to affiliate science or mathematics or other technical clubs with JETS or to organize and affiliate new chapters of JETS. State Headquarters furnishes suggested programs and a number of other services to the schools as briefly described in this paper. Cost for student membership, which includes admission to the competitive testing program associated with the Annual State JETS Conference, is $1.00 per year.

Engineers throughout the State, and particularly those of the Texas Highway Department, have enthusiastically joined the program and offered their help to any high school as engineer advisors to JETS chapters. The JETS program for a chapter in high school is fully under the control of the local school and under the guidance of a school sponsor, who is usually a Mathematics or Science teacher. The engineer-sponsor is available for help if called upon, but the program is under the full control of the faculty member who serves as school sponsor of the Chapter. Through the JETS program engineers and other professional people are encouraged to support the local schools in whatever way they can.

The State Headquarters for JETS furnishes the schools with a list of engineering scientists who have prepared presentations on timely subjects of interest to high school students and faculty, particularly those who have an interest in technological applications of Mathematics and Science. These programs are available without charge to the schools.

JETS sponsors the National Engineering Aptitude Search Testing Program, a testing program which is designed to measure the aptitude for the successful study of engineering. These tests were constructed by the Psychological Corporation of New York and the cost to the student is $3.00. The entire testing fee goes for paying for the test and, therefore, the State JETS office handles the administrative details of this program as a service to the student and his school. Consequently, counselors throughout the State are invited to contribute of their time to administer the test without remuneration. All high schools are invited to set up their own NEAS Search Center and administer the tests to their students. Whenever it is desired, the State Office for JETS will put the high school in touch with an engineer in the area who will volunteer his services to help administer the tests.

Members of the Texas Society of Professional Engineers throughout the State often volunteer their time as substitute teachers so that Mathematics and Science teachers may attend the annual conference for Advancement of Mathematics and Science Teaching in Austin. Also, these engineers provide this service so that teachers may attend the annual JETS Conference which is held on the Texas A&M University campus.

Many students and their teachers regard the annual JETS Conference as the highlight of the year for JETS activity. High School students from throughout the State compete in such areas as Mathematics, Chemistry, Physics, Slide Rule, Biology, Drafting, and Technical Writing. Appropriate awards are made to the winners and their schools. Also, several JETS scholarships are usually awarded.

The Annual JETS Conference includes a parallel program for Mathematics and Science teachers, and Counselors. This program is designed to encourage face to face communication between university faculty and high school teachers and to develop a better relationship between the teaching programs of the high schools and colleges.

Teachers and Counselors are encouraged to participate in this program and to submit their ideas on how the program can be made more profitable both for the student and the faculty. The JETS Annual Conference for the 1971-72 school year has been scheduled for March 17, 1972 on the Texas A&M University campus and the program planned will be of interest not only to Counselors, Mathematics and Science teachers, but also to English teachers. A printed program showing scheduled activities will be available in December of 1971. Those wishing additional information may write Texas JETS State Headquarters, College Station, Texas.

*Assistant Dean, College of Engineering, Texas A&M University, and State Coordinator, JETS.

TEXAS MATHEMATICS TEACHER
J. William Brown, Editor
Texas Council of
Teachers Of Mathematics
Woodrow Wilson High School
100 S. Glasgow Drive
DALLAS, TEXAS 75214

NON-PROFIT
ORGANIZATION
U. S. Postage
Paid
Dallas, Texas
Permit #4899