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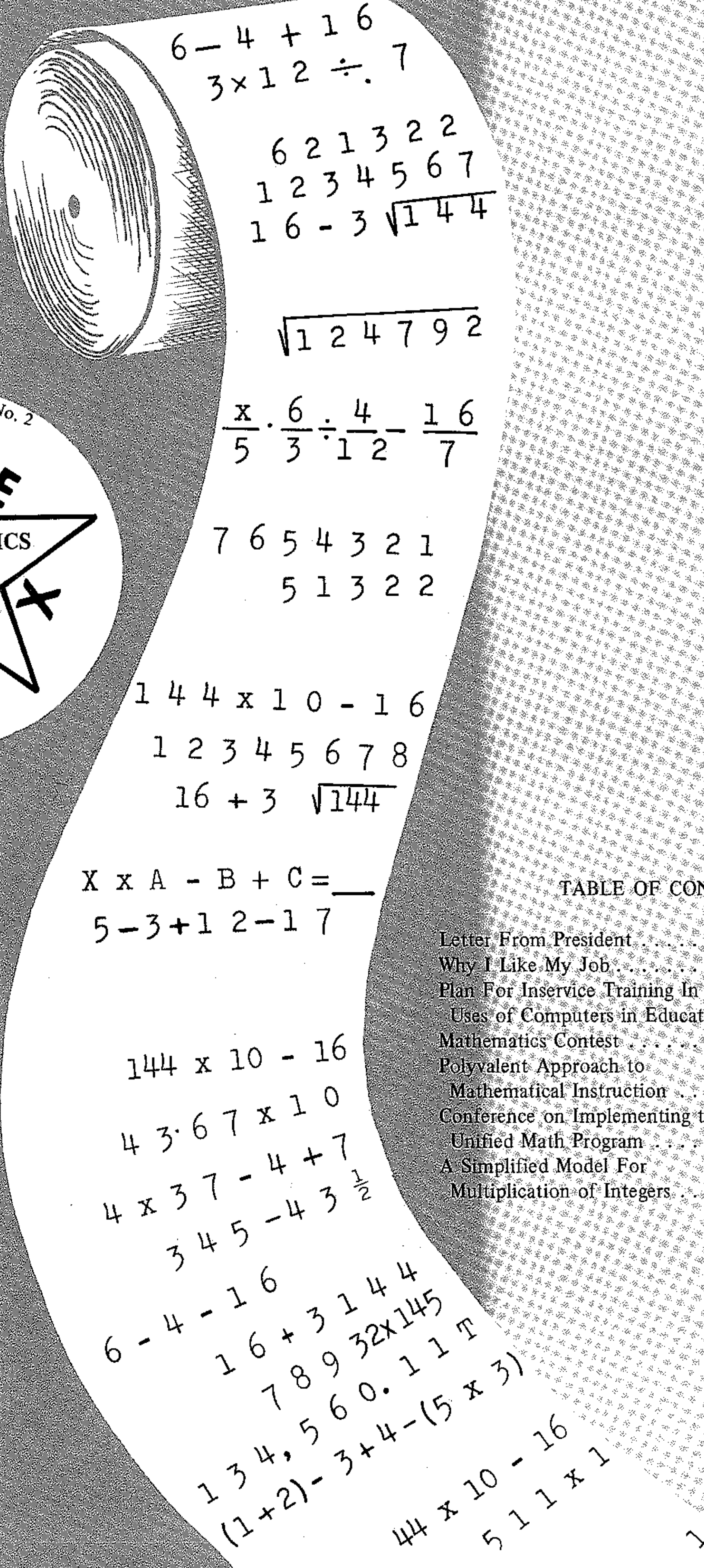

TABLE OF CONTENTS

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $2.00 entitles members to all regular Council Publications.

LETTER FROM THE PRESIDENT

Welcome to Mrs. Shirley Ray, President Elect, and Mr. Crawford Johnson as Third Vice President, as new officers of our organization. Shirley has been serving as our First Vice President for some time. I wish to extend a personal welcome to Mr. Crawford Johnson of Fort Worth Independent School District for his willingness to serve as one of our Vice Presidents.

I feel that the C.A.S.M.T. Convention was a success even though we seem to have some problems concerning the continuation of the Convention. I believe that the University of Texas people realize that it is not working too smoothly and they are going to make every effort to straighten out the trouble. They seem to want to continue to sponsor the Convention. We made it clear that if they didn't want to, the TASM and TCTM were ready to take the Convention and co-sponsor it. The mathematics part of CASMT is not to die under any circumstances.

By this time you should have received the Winter issue of the journal. The last report that I received was that we had 1320 members. This is the most members we have had for some time. Keep up the good work. Everyone must be a worker in order to keep an organization going. I hope that you are liking the journal better all the time and I must remind you that any member who joined after January 1, 1971, will be dropped from the membership rolls and will not receive a copy of the journal after the Winter issue. So join now.

The Executive Council voted in the meeting held December 10 to amend the Constitution and allow the amount of dues to be set by the Executive Council as the need might arise. They also voted to extend membership to institutional organizations that might wish to support the Council. This amendment should be forthcoming in the Summer issue of the journal and must be presented to the general body at least ninety days prior to the annual meeting. We recommend the next C.A.S.M.T. meeting be held in October.

We would certainly be pleased to hear from someone who desires to conduct a workshop this spring in the name of the Texas Council. If you would like to do this, please let us know and we are willing to help in any way that we can.

James E. Carson

We are still in need of articles for the journal. We know that you have something of interest going on in your school. Why not share it with others. Our journal has approximately 1500-1700 circulation at this time. We need your articles and we are interested in what is going on in your school. More and more open concept schools are being opened and they lend themselves well to the teaching of mathematics. Why not tell us how successful these schools are working out. Through the journal is one way you can communicate with other mathematics teachers throughout the State as well as getting the news out to all points of the United States. Write those articles!!

By the time this message is published we will have had a nice NCTM meeting in Beaumont. They have a fine program. Congratulations to the people who have had a part in making the meeting a success. I am looking forward to seeing many of you there.

WHY I LIKE MY JOB

CARL B. ALLENDOERFER

UNIVERSITY OF WASHINGTON

1. *Introduction.* Today is only a few weeks after the 34th Anniversary of the day I taught my first class—at the University of Wisconsin in Sterling Hall, the building which was bombed so severely last spring. I was a grass green Ph.D. who had never been a T.A., and I was mighty scared. On my mother's side I come from a family of preachers and college professors and so I looked forward to the life of a teacher; and now after the years of preparation, the big day had come—I was on the air at last. Somehow I managed to survive that first class, and since then I have never regretted my choice of a profession.

You, too, are teachers and together we share the many satisfactions of our jobs. Each September is an exciting time when we meet a new crop of bright-eyed, bushy-tailed youngsters who are eager to see what their new teachers are like. Then comes the first challenge—we must get them interested in learning mathematics. Most of our students are enrolled in mathematics because they are required to do so—to get into college, to learn tools needed for business, science, or engineering, or to fulfill some college requirement. All too often they have been turned off by a previous experience with mathematics and express their frustration with the common question "what good is all this?" It is a great satisfaction to me to be able to turn these students around, to show them that mathematics is full of beautiful ideas, to develop their intuition, and finally to convince them that they, indeed, can be successful with mathematics. To do this is hard work, for it means much more than just following the textbook, and I am not always successful. But working at it is great fun and the successes are extremely satisfying.

Even in these days of mass education with overly large classes, and impersonal and over-extended universities, a teacher who makes the effort can get personally in touch with most of his students. You can follow your students to the university, and I follow mine in their graduate work and later in their careers. I have former students all over the country who write me to ask for help, to tell me what they have been doing and occasionally to call on me to chat the "good old days". To be able to point young people in the right direction and to watch them succeed makes life very worthwhile.

2. *Development of Mathematics in my time.* All these satisfactions are true for teachers of all subjects, but mathematics in the past 25 years has been an unusually exciting occupation. On the one hand, new mathematics has been discovered at a very astonishing rate, and on the other hand more and more students at all levels have needed to study mathematics. The major problem for mathematics teachers has been to bring as much as possible of the spirit and content of modern mathematics within the range of our students, bearing in mind their backgrounds and abilities.

Their chief difficulty in doing this has been to bridge the communication gap between the active mathematicians on the frontier and school and college teachers in the classroom. The difficulty of communication is one of our most serious impediments to the solution of our social problems, and it has been no less serious in mathematics. Indeed communication was minimal in the period between the two world wars when there was very little contact between research mathematicians and college teachers, and between college teachers and school teachers. The most encouraging development since World War II has been the bridging of this gap. Just as one example, I mention Professor Hassler Whitney, one of the most creative mathematicians of his generation, who is now spending his full time on the problems of elementary school mathematics. Other examples like Project SEED have shown the invalidity of the old bromide "Professors may know mathematics, but they don't know kids." If the professors are smart and make the effort, they can get to know kids as well as mathematics.

It is this effort at communication which has led to the misnamed "New Math." The New Math has many faces and many versions, but when it has been properly formulated it has always involved these four considerations: (1) The development of intuition so that the abstract grows out of the concrete, (2) There is emphasis on the ideas of mathematics, (3) The reasoning behind computational rules is carefully explained, and (4) Computations and applications are given the necessary attention and time. Balance among these should be our watchword.

As in any revolution we have our zealots who have gone too far and too fast, and thus have harmed the movement. As with our political radicals some have sought to destroy the old before the new was ready to take its place. This has led to controversies and counter-revolutions. But so is the anatomy of any revolution. Fortunately the moderates have largely prevailed. I am, however, concerned about two trends that I believe to be unfortunate.

(1) The gradual deterioration of geometry in the school curriculum. Rather than throwing in the sponge and watering down geometry, we should strengthen it. More (not less) informal geometry is needed in the elementary and junior high schools, and thus time can be provided for coordinate geometry and solid geometry in the high school. No student can be successful in more advanced mathematics without an adequate knowledge of geometry.

(2) The pell-mell rush into calculus whether the students are ready or not. We are badly warping our high school and college curricula by shortening the pre-calculus training of our students. As a result instruction in calculus must proceed at an ever slower rate with the omission of more and more important topics. I do not object to quick courses in calculus for people like business students who need only its rudiments, but I am unhappy when I find the standard course going down hill because the students in it are just not prepared for it.

I have greatly enjoyed spending time on the New Math, and I hope that I have helped to keep it on a middle course between the extremes of over-abstraction and mindless manipulation. At meetings like this it is heart-warming to find so many teachers who have studied out of one of my books or who have taught one of them. Most everyone who speaks to me is very polite, but sometimes reactions are less friendly. A year ago I got a letter from two high school students in New England who urged me next time to write a book in *English.* Apparently the publisher slipped and sent them the Spanish or Japanese or Yugoslav translation! But such comments are rare. I have two students this year in a graduate class who became interested in mathematics when they studied one of my books in school. Now they are faced with double jeopardy!

3. *What Lies Ahead?* It seems to me that the New Math has reached a plateau. The new texts are in print, SMSG is closing down, and there is danger that we are drifting into a new orthodoxy. Since the old revolutionaries are either dead or losing their zeal, new leaders must emerge. Groups like this are the places where they must be found. There are three important directions that may take the stage in the seventies.

(1) We must adapt our teaching to the computer revolution, for all our young people (like it or not) are living in the shadow of Babbage, the inventor of the computer. It is important to understand that programming a computer is not mathematics, but that computers help mathematicians do the computational portions of the solutions of their problems. Now that computers have replaced logarithms as the best means of doing fancy arithmetic, we need to shape our mathematics to take advantage of these modern methods. For example, many problems in integration and the solution of differential equations are best handled by computers instead of by textbook methods.

I expect a major revolution in the teaching of calculus as soon as mathematicians awake to the presence of the modern computer.

(2) New methods of teaching are rapidly descending on us, and we must learn to take advantage of these. In the last analysis, students really learn by themselves and not from teachers. The function of the teacher is to inspire, to help out in cases of trouble, and to guide the student along the proper path. The teacher should not be the main source of information, but should refer the students to books, films, etc. to get his knowledge. The lecture method of teaching is long since dead, but we are having a terrible time giving it a decent burial.

So we must adapt to the new means of conveying information. I have been producing mathematical films for this purpose, and have just published a semi-programmed, self-study, textbook in arithmetic and geometry for prospective elementary teachers. More mathematicians need to get into this business.

(3) More emphasis will be placed upon the performance of school systems in their instructional programs. We will be asked to state in agonizing detail the objectives of our courses, and our performance will be judged according to how well our students meet these objectives. As with all innovations, this cult of performance may go too far and assume ridiculous proportions, but there is substantial merit in it. It is our job to make use of what is good in this field and to discard the rest.

4. *Conclusion.* I hope that I have conveyed to you the excitement and satisfaction that I find in my job as a teacher of mathematics, and I am sure that you have similar feelings yourselves about your own jobs. We are lucky to be able to earn our livings doing many of the things that we like best to do. We have our trials and tribulations, to be sure, we are the fortunate ones. Because of our good fortune to be mathematics teachers let us work even harder for the good of our students and our profession.

DALLAS NCTM
AREA MEETING
March, 1971

PLAN FOR INSERVICE TRAINING IN USES OF COMPUTERS IN EDUCATION

JAMES E. LEININGER

Objectives–

The primary objective is to acquaint teachers with the many possible uses of computers in education. It is hoped that the presentation will help the teacher realize that the computer cannot replace books, much less teachers, but can serve as a powerful ally. It can relieve the teacher of much routine clerical work and repetitious drill with students. It is hoped that the teacher will be inclined to accept computer help as it becomes available.

Student Activity–

This is one of several inservices envisaged to acquaint teachers with alternative teaching methods by making use of the old cliche–a teacher teaches as he was taught. It is hoped that by demonstrating these new techniques can indeed be successful, the teacher will be encouraged to try them in his own classroom.

This particular inservice is to acquaint the teacher with computer tutorial capabilities. In this, a new concept is introduced by the computer which then proceeds to help the student acquire skill in its use. The subject of this tutorial is the use of computers in education. The teacher will be required to take time during one of his planning periods or on an inservice day and go through the program on an appropriate device. Teletype units which can communicate with the Region IV computer are available in all HISD high schools. The computer program necessary to this inservice is no practical barrier, although I imagine its creation would involve a prodigious amount of work. This program could be written by students however.

The teacher will report to the console and will be shown how to operate it. He will then be left alone with it. The session will start with the machine asking the teacher for his name and what he teaches. The first is important because the computer will address the teacher by name, an act which has been proved to make the computer seem less impersonal. The second item is important because examples will be taken from the teacher's field whenever possible to strengthen the presentation. The computer will then talk about how computer aided instruction can be used in that field and will ask questions of the teacher so it can pursue his interests.

In the course of the session much of the following information will be passed to the teacher. First, the number of clerical tasks which the computer has taken from the teacher will be listed. Then, it will talk about its potential role as a teaching aid. The computer can conduct the repetitious drill and answer the questions which most students ask leaving the teacher free to handle the special difficulties which so often must be neglected because of lack of time. Sample questions from the teacher's field will be given. A computer can currently be working with 200 students at once through time-sharing programs; each student at a different point in the material or even in a different subject. It is likely that soon computer will be able to handle 2,000 students at once. It can scale the difficulty of the drill to the ability of the student, skip material or go over it patiently a hundred times and all the while it is giving the feedback needed to improve performance. The computer itself is a very powerful tool which is available to the student to explore with and learn. It is a good bet that if a student can program a concept on a computer, he understands the concept well.

The computer will then try to allay some of the fears teachers might have about computers. Don't students find this computer aided instruction impersonal? It has been demonstrated that with gadgets like tape recorders and slide projectors it can be made more personal than a lecture or even a recitation section. Isn't it difficult to get programs suitable for your classes and more difficult to write them yourself? To get a program covering a specific area is perhaps difficult. To write them yourself requires only a little more effort than to write a multiple choice test. The computer in addition will keep track of how your students do on the individual questions. This allows you to find unreliable questions and your programs can be constantly revised and improved. In addition, since most courses in the same subject are similar, it would be easy for a large number of people to share a library of programs. Aren't computers going to take over and replace me in my job? A computer can do anything that can be reduced to a series of simple instructions. It is not likely that system analysts will become proficient enough to reduce the many faces of teaching to a computer code. Teachers will be needed as much as ever to do those things computers can't and which they will finally have time for. If by some development teachers should be replaced, we get some consolation from the fact that so will nearly everyone else.

Finally the computer will offer the teacher reprints of articles on computer uses in the teacher's field, take any requests, and say goodbye.

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MATHEMATICS CONTEST FOR AREA STUDENTS

HOLLIS I. COOK
West Texas State University

During the spring semester of each school year the Department of Mathematics at West Texas State University holds a Mathematics Contest for junior and senior mathematics students from the surrounding area high schools. Students from 35 area towns have attended the contest in the past three years. One purpose of the contest is to give area mathematics students an opportunity to become better acquainted with West Texas State University and its mathematics teachers. Another purpose of the contest is to give these students an opportunity to compete with one another in a mathematics contest.

In addition to the competition of the contest itself, the students are given an opportunity to visit the various buildings on the WTSU campus which include an outstanding museum and a fabulous student activity center. The student is also given a chance to attend counselling sessions on various job opportunities relating to mathematics such as engineering, computer analysis, and the teaching of mathematics. Individual counselling sessions for prospective students can also be arranged at the request of the individual student.

The contest itself consists of questions concerning algebra, plane geometry, and basic logic. Questions involving trigonometry and annlysis are not included in the contest, in order that each student will have studied all of the areas from which the questions are chosen. In an attempt to make the contest as fair as possible to each student, the students compete only against other students from schools in the same interscholastic league classification. Thus, a first, second, and third-place winner is announced in each of the five interscholastic divisions.

During the past three years, winners in class 4A have come from Amarillo High School, Tascosa High School of Amarillo, Borger High School, and Pampa High School. Winners in class 3A have come from high schools in Canyon and Tulia. Winners in class 2A were from Childress, Phillips, and Shamrock. Winners in class A were from Silverton, Springlake-Earth, Kress, and Alamo Catholic High School of Amarillo. Winners in class B were from Wellmon, Adrain, and Lefors.

Polyvalent Approach to Mathematical Instruction

It is fashionable in these days of rhetorical excess *to describe change as revolutionary in scope.* The mass media remind us daily that revolutions are occuring right under our noses. We hear of (and see) the *Social Revolution,* the *Sexual Revolution,* the *Technological Revolution,* the *Student Revolt,* the *Faculty Revolt,* and so on. *Apparently any complete or sudden change* in the conduct of human affairs, with or without a violent confrontation or an exchange of power, *may properly be called a revolution.*

It is predicted that the *last three decades of the twentieth century* will witness *a drastic change in the business of providing instruction in schools . . .* You will note the *omission of words like "teaching" and "learning"* in describing the coming revolution. At the present time, teaching connotes the vision of the *"stand-up" lecturer.* It is likely that future terms for teacher may be *"instructional agent"* or *"lesson designer"* or *"instructional programmer."* As for *learning,* it is believed that the word will not be a way of describing an *activity* of the student, but rather *a way of characterizing change* in the student's behavior in some desired direction *between two definite time markers.*

Some pupils learn content rapidly; others learn it more slowly. This fact accounts for educators' *preoccupation with rate of learning; how much, how fast?*

More recently new proposals have been finding their way into schools; *elimination of grade levels in elementary schools; team teaching* and employment of *paraprofessionals and teacher aids; programmed learning and learning laboratories; and acceleration and enrichment.* The implication that standard content should be learned more rapidly has been evident in most plans. Yet, educators realize that rate of learning is only one consideration. Learning has numerous dimensions; above all it is personal.

Valence in chemistry means the combining ability of a given element with another element under a given environment as in oxidation and reduction phenomena.

The mathematics teacher faces a polyvalent environment (climate) as such teacher goes from one class to another. The sample space consists of the thirty (more or less) different personalities making up the classes. Just as many chemical elements exhibit different physical characteristics depending upon the excitation (or lack of it) of its environment, so each student reacts polyvalently from day to day (even hour to hour) depending upon the sample spaces provided in the summation of their different climates of environment.

Not only is the student affected by his physical environment (including home and school), he is affected by his cognition (cognitive ability) and to a greater extent by the events making up his field in the affective domain which is largely the result of interaction with parents, classmates, teachers, and the social struggle surrounding him.

There are many who think that "individualizing" education is a means for protecting against such an ambush by those who cling to the "customaries" of the past. *Two of the most notorious procedures for "individualizing" instruction have been flexible scheduling and computer-assisted instruction (CAI). In flexible scheduling we find the same people doing the same high-gloss nothing on a different time schedule.* Substituting *"module" for "class" simply won't do the trick;* neither will lecturing to 15 instead of 45. As for CAI, the only significant "individualizing" that it provides is a flexible rate for each student to learn the same thing. *CAI programs are, in the main, electronic textbooks still operating on a Yes-No pattern paradigm teaching that people can be wrong but machines cannot.* These innovations are immature and at the moment tangential to what we feel is the significant problem.

We *need to "individualize" education, but even more we need to personalize it.* We deal with little people and big people who care about each other and about learning. We trust that people will learn unless they are prevented from doing so. *We assume that a child will learn new things naturally, by himself, asking the questions he needs to,* and *that adults should help show him the way* and then get out of it. *The aim of a good teacher is to become unnecessary. Not many of us can stand that kind of ego torpedo, and that's one reason why not many of us are good teachers.*

To become unnecessary, we mean that one *is not* to *abandon a young person who is yet without self-confidence, an understanding* or *personal responsibility,* a *sense of trust in others,* or *a secure willingness to risk and to explore.* It is rather *to help create these qualities, to replace coercion and control with consent, to replace fear and dependence with trust and cooperation.* Everybody with even tepid blood knows this, but surprisingly few know that such things cannot be taught. *They must be experienced to be understood. This means that a teacher must provide the opportunity and the environment for such experiencing if he wants to be a good teacher, i.e., eventually to become unnecessary. It takes a person who is secure and open about himself and his job as a teacher to do this. That's why it is not done by many nor often even by a few. It is a risky thing to do, and we are conditioned cowards. It might be well to remember that cowardice - - - and so is openness, security, and the willingness to risk.*

We are *talking of personal meaning, a level of knowing that is deeper than information inquisition. These days, the transmission of information can be done* in a variety of more efficient ways than the traditional lecture-by-mouth method. The *discovering and sharing of personal meaning,* however, *is a matter of individual confrontation and communication.*

Let us go, for a moment, to an analysis from chemistry again. Iron, a chemical necessity in today's technological world, is polyvalent. Ferric iron, of valence 3, is a beautiful yellow color when in solution as ferric sulphate having been excited to this condition by a favorable environment such as postassium per manganate becomes ferrous iron of valence 2 when in the environment of a reducing agent and thus changes color from the previous bright sparkling yellow to a brilliant green. Iron has thus exhibited a polyvalent property.

Our mathematics instruction, to be most effective, is improved by our ability to recognize that some students are at the ferrous state (green) and can be brought to the ferric (yellow) state by suitable excitation. One student may have only the ability to be a modest operator in mathematics; another may have the capacity to be a rapid operator if furnished the invented tools of the mathematician, appropriate rules condensed to formulas, another may have the capacity and desire to do deductive reasoning; and lastly, a few may have the spark of genius to be a creator of mathematics. All of these students learn more rapidly and retentively if given excitations consisting of reasonable successes, being held in high esteem by the teacher as an individual worthy of acceptance, and accepted by class members. The teacher has the God-given opportunity to be an exciter for learning by providing learning opportunities that fit the valence needs of the student.

Many of our authorities in the instructional field today feel that an association of topics geared to the learning rate and learning style of the student and, at the same time, made relevant to him through developed applications may serve as catalytic agents for cognitive excitation for learning the mathematics that is most satisfying to the individual. Without inner satisfactions on the part of the student derived from the total learning process, the student at the end of a given interval of time will little have learned and less remembered.

Teachers have a need (some say duty) to monitor achievement, to let the student know how he is doing. This function, however, should not become a prime issue either in time or classroom focus, and it should never act as a deterrent to motivation. It should be handled efficiently, it should provide comprehensive feedback for each student and the teacher, and it should be a valuable resource in itself.

Two Basic Ideas

Continuous random sampling. In our model, continuous random sampling techniques are applied both to the learners and to the final objectives of the course being taught. *Instead of testing everybody every week on the same items of a segmented unit of a serial-step design,* it is *proposed that a sample of students be tested at randomized intervals on a randomized selection of the final list of performance criteria, or course goals.* The data provided by this method are much easier to handle, take less of the teacher's time, less of the student's time, and can be used to establish individual learning curves for each student, teacher, course, and teaching method over as long a time period as desired.

Continuous feedback in terms of final performance criteria. The *first point* to be made is *that the final performance criteria* (PC) *(OR BEHAVIORAL OBJECTIVES) must be established at the beginning of the course.* This *can be done by department, by individual teachers,* or, preferably, *by the teacher and the students cooperatively.* It amounts to *sitting down beforehand and figuring out precisely what is to be accomplished over the course of a week, a term, or a year. PC or BO must be specific and precise if they are to be effective. They must be stated in such a way as to be clearly understood by the students in terms of objectively defined terminal behaviors marking the achievement of the PC or BO.* In other words, *the teacher should prepare a list of specified behaviors which students must be able to perform by the end of the course. This list should be given to the students and be subject to revision, modification, and final acceptance by them. When everyone knows precisely what is expected of him, much time and frustration will be saved* - - - the *teacher won't have to "hide" test items* and the *students won't have to "psych out" the teacher to discover what they are supposed to know.* Also, the final examination, the list of PC or BO, can be given to students in the first week so the teacher and the students can discover how much they already know about the course material, thus avoiding a great deal of redundancy and enabling the students to share with each other their varied stores of knowledge.

The *second important point* to be made is that *"feedback" is not the same as "evaluation."* We have found that *evaluation tends to separate the judge and the judged;* it *drives them away from each other and makes meaningful communication much more difficult than it would be otherwise.* Evaluation also tends to be manifested in the form of extrinsic reward, which serves more of a failure-avoidance function than a positive motivating function. *Feedback,* on the other hand - - *merely letting the student know how he is progressing toward the goals he has selected - - is a non-evaluative process because it does not involve a grade. It is just information about where the student is now in relation to where he wants to be.*

Now, when these two concepts are put together, several things happen. *Testing,* or information transmission, *need no longer constitute a prime issue either in class time or class focus. Not everyone takes the tests at the same time and not everyone takes the same tests, so there is no reason to prime the whole class through the week for the climax on Friday or some other day. Everybody is at a different stage in terms of the final PC or BO;* hence, *there are varied foci of interest among the students.* There is no point in lecturing to these students, but *it is very important to answer their questions and respond to their need for resource help.* There is *no anxiety-provoking test* anticipation to create a need for frantic study on material for a given test (better called a feedback monitor). *The random sampling of students and*

PC or BO items provides a comprehensive and highly efficient feedback system for the teacher and the students in a very efficient way. Less time is spent taking and marking tests; the results can be used by other students as resource material for their own progress. (In this way the feedback monitor becomes a learning experience in itself and thus a valuable resource.) Since *there are no grades attached to the feedback,* there is no need to covet correct answers. *Cooperation replaces competition. The point is for everyone to reach the minimum PC or BO level, to "succeed" in the course. If a student arrives at the minimum level of achievement during the second week, he has passed the course* and is essentially free to go on with whatever interests him further, or, if he wishes, to share his knowledge with others in the class. He need not just sit around waiting for the months to pass. It is assumed that if a teacher gives a list of PC or BO to his students at the outset, he may not have to do much more except answer questions about resource material - - the teacher begins to *become unnecessary.* When entrusted with the responsibility for their own progress and relieved of the constant threat of judgment, the *students in effect take over the information-gathering tasks that the teacher has traditionally assumed. Class time is liberated for other things;* namely *exploring personal meaning.*

Our most recently built high school, the Skyline Complex of Dallas, has advanced clusters or areas of learning which we can call Career Development Centers. The mathematics for this fact of CDC is listed as Advanced Mathematics. The originator of the concept for the mathematics areas has as a goal the ideas of collecting data, organizing data, and interpreting data. Before you say that this goal is somewhat unsophisticated, let me remark that practically what everyone does in making decisions has inherent in it these elements which constitute the basis for statistics. Actually, all mathematics is served by a statistical organization. When you solve a mathematical problem do you not mentally collect the data given in the problem, then organize these data in a manner served by your process of deductive reasoning and lastly interpret the processing used by serving up a solution or answer?

The CDC student enrollee, though not necessarily an honor student, must have proven himself a capable student and have been at least moderately dedicated to the purposefulness of learning mathematics. The classes will prove to be made up of polyvalent individuals. The student will be processed to begin advancing in mathematics from his initial mathematical maturity through mathematical topics geared to fit his manifested ambitions for a career. He will not only visit the institutions or businesses which seem more likely to suit his career taste, but will have been encouraged to participate on a non-paid working basis with an individual or individuals associated with the concern of his choice.

J. WILLIAM BROWN
(Speech Made in Austin;
Some Ideas Borrowed)

CONFERENCE OF IMPLEMENTING
THE UNIFIED MATHEMATICS PROGRAM

Southern Methodist University announces a National Science Foundation Conference for Development of Resources Personnel to Implement a Unified Mathematics Program, 7-12, developed by the Secondary School Mathematics Curriculum Improvement Study, to be held at SMU, Dallas, Texas, August 14, through August 18, 1972.

The SSMCIS program is designed for the upper 15 to 20 per cent of the school population in scholastic ability. The program presents mathematics from a contemporary point of view. Based on an informal study of sets, relations, mappings, and operations, significant mathematics (traditional and modern) is organized around the structural properties of groups, rings, field, and vector spaces; the realizations of these structures (number systems, matrices, geometries); and the activities and applications found in algebra, geometry, probability, linear algebra, computers, numerical analysis, and the calculus.

The purpose of the conference: (1) to acquaint college and secondary mathematical personnel with the philosophy, objectives, and content of the SSMICS program; (2) to develop college and school cooperation in innovating the program; (3) to develop leaders who, after teaching the program, can serve as in-service trainers of secondary school teachers in the background, content, and methods of teaching a unified mathematics program; and (4) to secure innovation and experimentation with the SSMCIS program.

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A Simplified Model for Multiplication of Integers

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To be effective, any mathematical model must be composed of elements that are familiar to the user. The use of any model should lead to an abstraction so that the user becomes free from the constraints usually present in using models.

Most of us use a number line as a model to illustrate addition and multiplication of Whole Numbers. We speak of taking trips on a number line and of repeated trips on a number line. If a model can be developed using trips on a number line for the operations with integers, we may benefit from a "carry-over" from the operations with the Whole Numbers.

A model using the number line to illustrate addition of integers is fairly well known. What remains to be done is to construct a model for multiplication of integers in which the number line is used. We first explain briefly the model for addition of integers. This particular model involves an East-West street and the taking of "trips" on this street. In the illustration below, we call this street Central Avenue. If we are located at the zoo, [Z], and wish to go to the museum, [M], we must take a trip of two blocks to the East. We denote this trip as "$\overrightarrow{2}$" (read "right-two").

If we then wish to go to the bank, [B], from [M], we could proceed from the museum along Central Avenue 5 blocks to the West, denoted "$\overleftarrow{5}$" (read "left-five"). In order to get from the library, [L], to the coliseum, [C], we would take a trip denoted "$\overrightarrow{4}$".

Let us assume that we wish to visit several attractions along Central Avenue, and that we also wish to know *how far away, and in what direction, we are from the starting point.* If we start at the jail, [J], and proceed to the zoo, we denote the trip as $\overrightarrow{2}$. If we then go from the zoo to the coliseum, [C], we take another trip of $\overrightarrow{3}$. We see that, *from the beginning of our first trip to the end of our excursion,* we are 5 blocks to the East of the jail, denoted $\overrightarrow{5}$. Hence we see that the two distinct trips, $(\overrightarrow{2}, \overrightarrow{3})$ is equivalent to taking a single trip of $\overrightarrow{5}$. We then associate $(\overrightarrow{2}, \overrightarrow{3})$ with the equivalent trip, $\overrightarrow{5}$:

$$(\overrightarrow{2}, \overrightarrow{3}) \Longrightarrow \overrightarrow{5}.$$

We see that each of the symbols, "$\overrightarrow{2}$", "$\overleftarrow{7}$", "$\overrightarrow{6}$", and so on, tell us two things:

A. A distance, or magnitude (absolute value), and
B. One of two opposite directions.

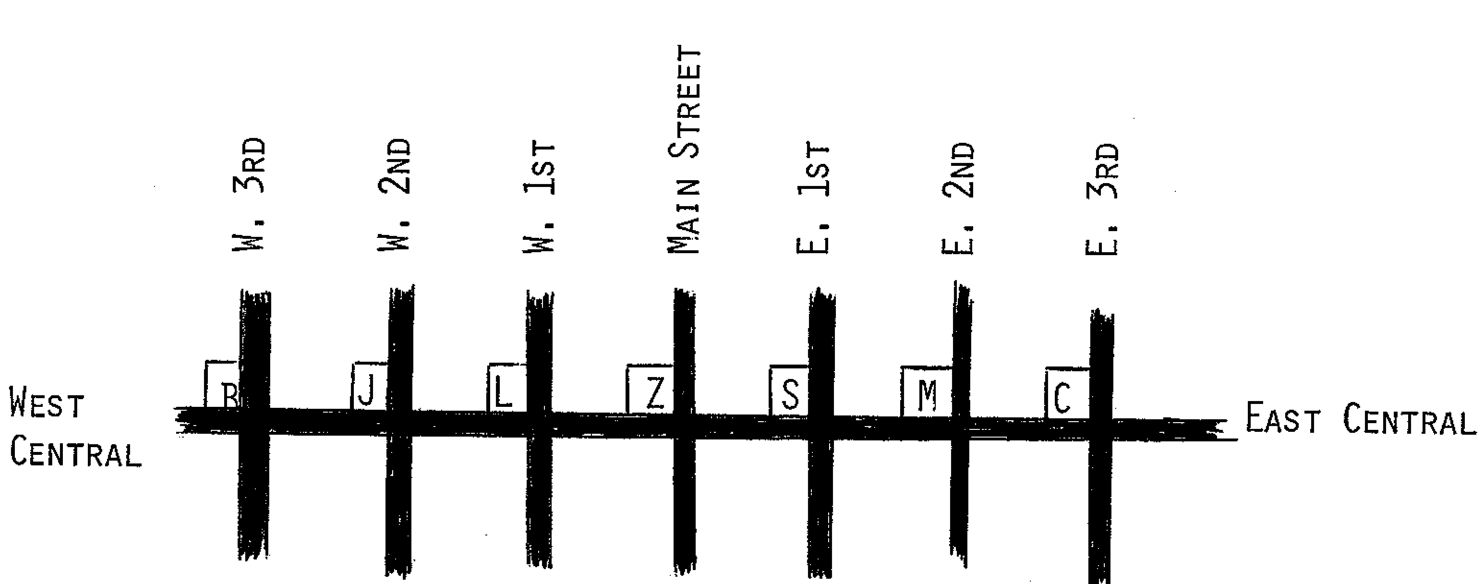


Figure 1.

Additionally, if we take several consecutive trips along a "line", we are interested in "what single trip tells how far and in what direction we are from the beginning of our first trip to the end of our last trip:"

$$(\overrightarrow{7}, \overleftarrow{4}) \Longrightarrow \overrightarrow{3};$$
$$(\overleftarrow{10}, \overrightarrow{16}) \Longrightarrow \overrightarrow{6};$$
$$(\overleftarrow{4}, \overrightarrow{3}, \overleftarrow{6}) \Rightarrow \overleftarrow{7};$$
$$(\overrightarrow{14}, \overleftarrow{2}, \overleftarrow{8}) \Rightarrow \overrightarrow{4}.$$

The numbers which we have called "right numbers" (Ex: $\overrightarrow{10}$), or "left numbers" (Ex: $\overleftarrow{7}$), are generally referred to as positive integers or as negative integers, respectively, and the idea of a pair of trips, one of which is "tacked onto" the other provides an excellent model for adding integers. Using this model for addition, the trips

$$(\overrightarrow{4}, \overrightarrow{7}) \Longrightarrow 11$$

become $\quad {}^{+}4 + {}^{+}7 = {}^{+}11.$

The numbers which we have called "right" or "left" numbers can be placed in a one-to-one correspondence with the positive integers or the negative integers, respectively. If we consider the *integer number line,* we can write "${}^{+}4$" instead of " $\overrightarrow{4}$ " to name positive numbers.* The numerals "${}^{+}4$" and "${}^{-}4$" may be read as "positive four" and "negative four", respectively.

We must agree that we use positive numbers to measure observed changes to the Right and negative numbers to measure observed changes to the Left. Hence, we can think of the sum

$${}^{-}9 + {}^{+}5$$

as measuring the resultant change from the beginning of our first trip to the end of our last trip. For simplicity in interpreting the resultant change, each trip will begin at O. The example ${}^{-}9 + {}^{+}5$ is illustrated below:

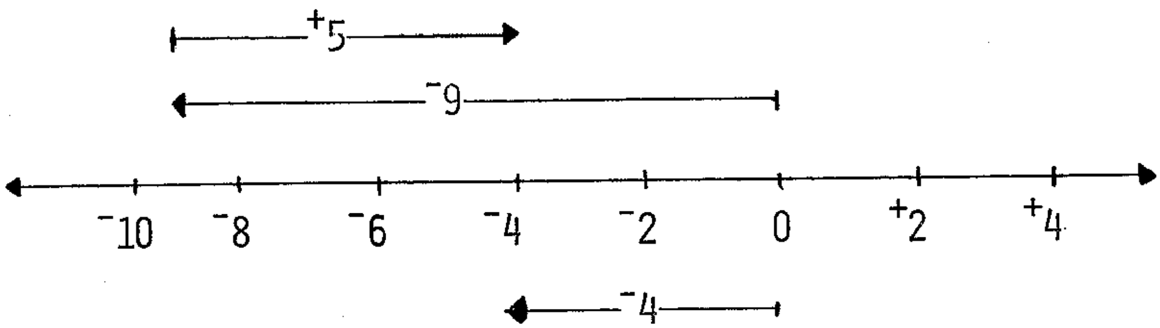


Figure 2.

From the above, we see that the resultant change is ${}^{-}4$, or

$${}^{-}9 + {}^{+}5 = {}^{-}4.$$

When we consider multiplication of Integers, we find that in one particular instance, such as

$${}^{+}3 \times {}^{+}4 = {}^{+}4 + {}^{+}4 + {}^{+}4$$
$$= {}^{+}12.$$

has a meaning which is exactly the same meaning as that of multiplication of Whole Numbers. However, since *each* Integer denotes a change in position by a certain amount and a change in one of two opposite directions, the usual definition for multiplication of Whole Numbers,

$$A \times B = \underbrace{B + B + \ldots + B}_{\underline{A}\text{ Addends}},$$

has no meaning when we consider sentences such as

$${}^{-}4 \times {}^{+}3 = \blacksquare \quad \text{or}$$
$${}^{-}4 \times {}^{-}3 = \blacktriangle \ .$$

Therefore, we need still another model for multiplication of Integers *which retains some properties of the previously studied model for addition of Integers.* To construct this model, let us consider a road with the two opposite directions, Right (R), and Left (L). Furthermore, if we think of a car on this road that moves to the Right, we can say the car moves in a *Positive* direction; if the car moves to the Left, then the car moves in the *Negative* direction, as illustrated in Figure 3:

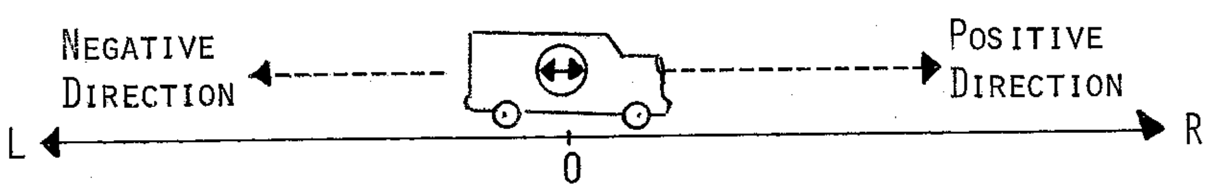


Figure 3.

*We will not consider trips of magnitude zero in this article.

We can think of the car being capable of moving backward or forward which is determined by the forward gear, “  ”, or the reverse gear, “  ”. If the car is facing R, in forward gear, and moves at a rate of 2 m.p.h. for 5 hours, we observe that the car would move to the Right a distance of 10 miles. This is illustrated below:

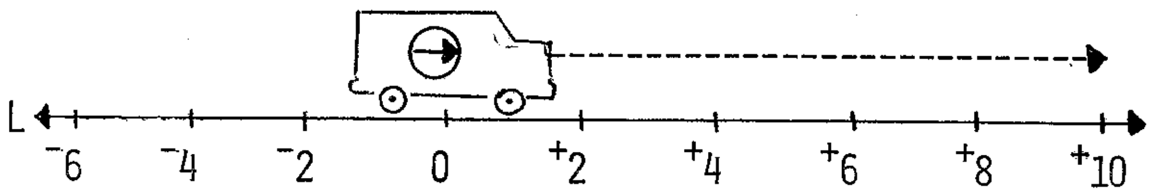


Figure 4.

Similarly, if the car is facing R, in *reverse* gear, and moves at 3 m.p.h. for 4 hours, we observe that the car moves to the Left a distance of 12 miles:

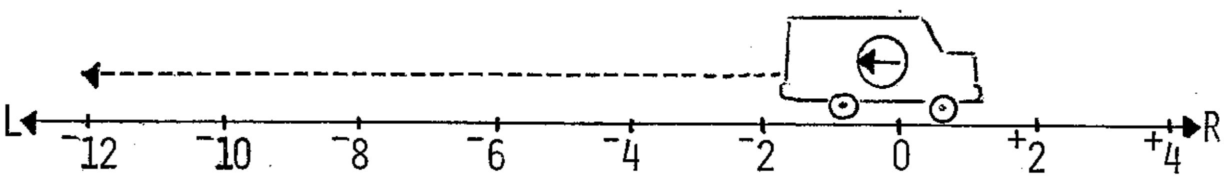


Figure 5.

In order to predict the resultant change in position of the car, you need to know whether the car is facing R or is facing L. Suppose the car is facing L. Will the car necessarily move L? The answer is “no”, because we do not know if the car is in forward or reverse gear. If in reverse, the car moves R; if in forward gear, the car moves L.

So in order to predict the resultant change of the car, you also need to know

A. The direction the car is facing, and a rate;

B The gear the car is in, and a measure of time.

Each of these things involve one of two “opposites” and a magnitude and hence can be denoted by Integers. We can apply these generalizations to examples:

A. If the car is facing R and moves at 4 m.p.h. we can denote this by “$^+4$”;

B. If facing L and moving at 3 m.p.h., we can denote this by “$^-3$”.

Also, we can use Integers to denote the gear the car is in and the length of time in this gear as follows:

A. If the car is in forward gear and travels for 5 hours, we can denote this as “$^+5$”; and

B. If the car is in reverse gear and travels for 5 hours, we can denote this as “$^-5$”.

How does this help in interpreting multiplication statements such as

“$^+5 \times {}^+3$” ?

We can think of $^+5$ as denoting the direction the car is facing and the rate the car is moving. We can think of $^+3$ as denoting the gear the car is in and the length of time the car moves. Finally, we can think of

“$^+5 \times {}^+3$”

as denoting *the resultant change in position of the car.*

In the case above, we think of the car facing Right and moving at 5 m.p.h. and in forward gear for 3 hours. So, the resultant change in position will be 15 miles to the Right, which is denoted “$^+15$”. If we think of the product of these two numbers as the measure of this change, we conclude

$$^+5 \times {}^+3 = {}^+15.$$

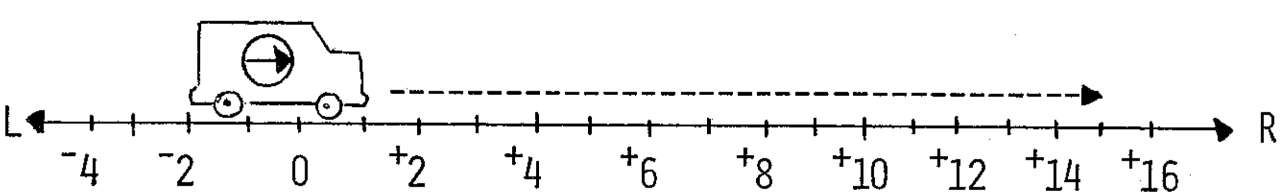
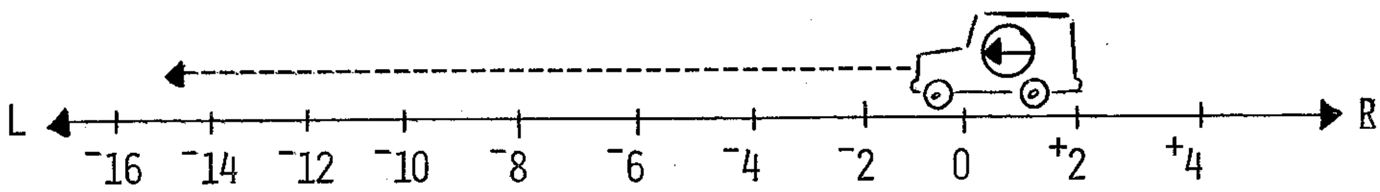


Figure 6.

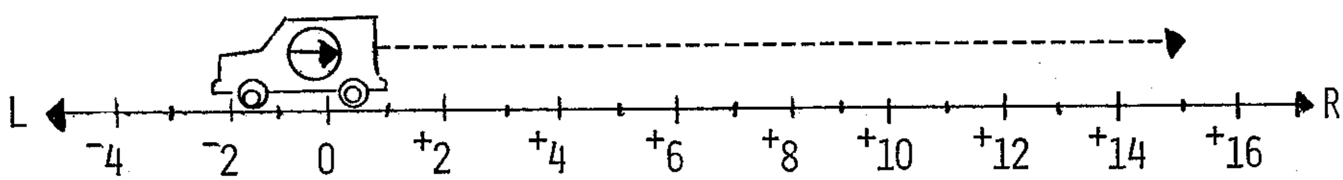
When we investigate the numerals "$^{-}5 \times ^{+}3$", we think of the car facing Left and moving 5 m.p.h. and in forward gear for 3 hours. The resultant change in position will be 15 miles to the Left, which is denoted "$^{-}15$":



$$^{-}5 \times ^{+}3 = ^{-}15$$

Figure 7.

If we wish to consider the sentence $^{-}5 \times ^{-}3 = ^{+}15$, we can think of the car as facing Left and moving 5 m.p.h. and in reverse gear for 3 hours. The resultant change in position will be 15 miles to the Right, which is denoted "$^{+}15$".



$$^{-}5 \times ^{-}3 = ^{+}15$$

Figure 8.

Hence, the model for multiplication retains some of the common elements of the model of addition of Integers, However, there is one extremely important difference between the two models.

First, when we consider the addition model, the concretization of both the addends and the sum was a change measured in direction and distance.

When we consider the model for multiplication, the concretizations of the first factor, the second factor, and the product are all different. The first factor is associated with a direction and a rate of speed. The second factor is associated with the position of the gear and the measure of time.

The product, however, as in the case of the addition model is associated with a change measured in direction and distance.

It should be noted that subtraction may be considered as "a search for a missing addend" and that division may be considered as "a search for a missing factor".

We strongly emphasize that a model should be used only until the operations have been abstracted. This abstracting is accompanied by stating in mathematical language the rules for the operations.

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