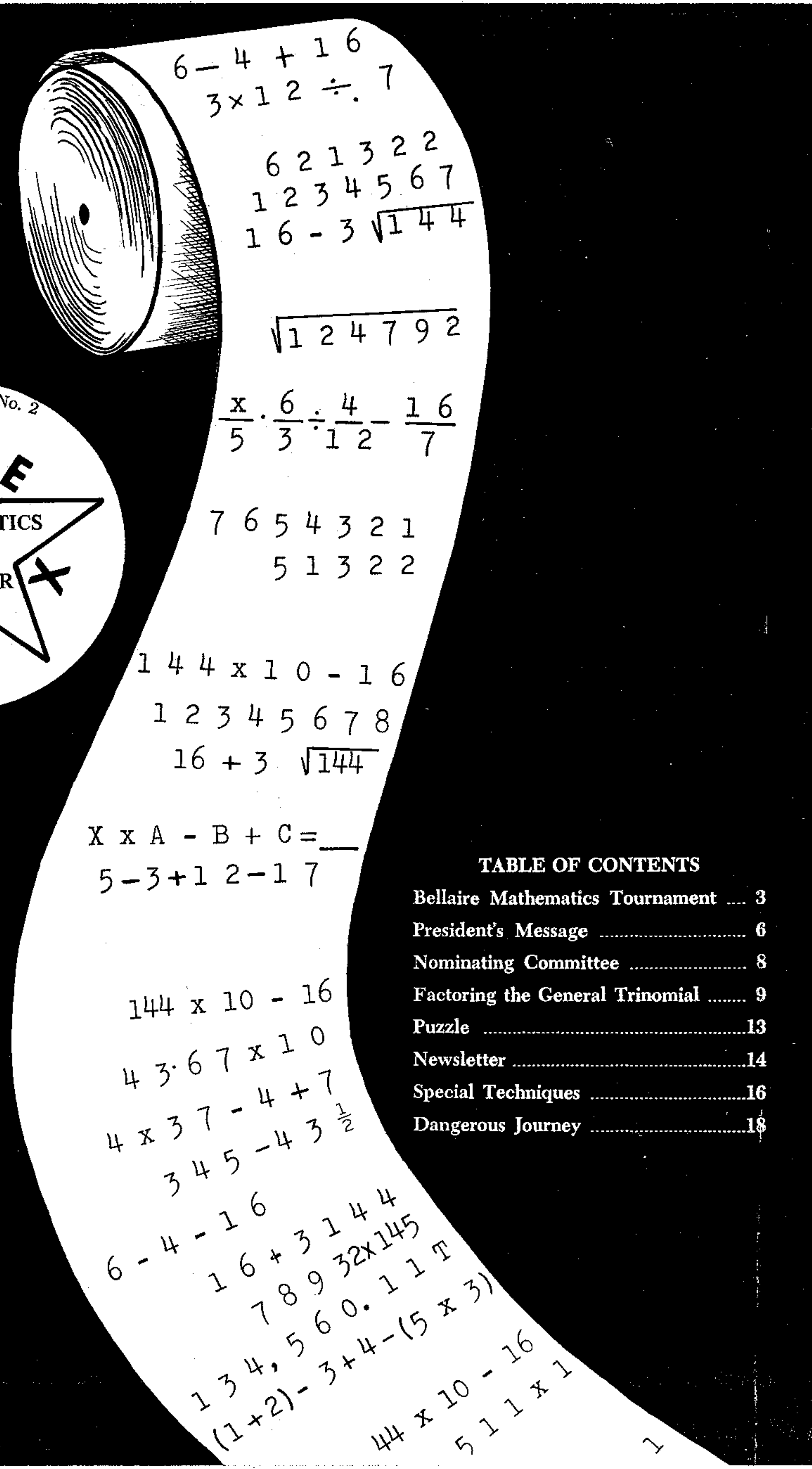
Volume XVIII, No. 2

TEXAS

MATHEMATICS TEACHER

Fall



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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. James E. Carson, *Texas Mathematics Teacher*, 3010 Bayshore Drive, Pasadena, Texas 77502.

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BELLAIRE HIGH SCHOOL ANNUAL MATHEMATICS TOURNAMENT

MISS RUTH GOLDBERG
MRS. MARY ANN ROBERTS
MRS. JAN GAYLORD

Background

The Mu Alpha Theta mathematics honorary society was organized at Bellaire in December of 1964. After several years of growth the organization became firmly established and wished to have an annual project. Consequently, the members planned and operated the first mathematics tournament at Bellaire in the spring of 1968.

Bellaire students wanted this tournament to be unique. Prior to this time the two tournaments in this area were limited to a few students from each school and prizes were awarded only to individuals. Bellaire's tournament, it was decided, should be for a larger number of students and awards should be on a team basis. With these ideas in mind, research eventually led to several articles[1] in "The Mathematics Teacher" which helped formulate the details of the contest. Eight schools attended that first contest but since then its popularity has grown and at this year's recent 4th Annual Tournament 38 schools were present.

[1]R. J. Mattson, **Mathematics leagues: stimulating interest through competition,** The Mathematics Teacher, Vol. LX, No. 3, (1967), pp. 259-261.

T. J. Paarlberg, **The Mathematics league,** The Mathematics Teacher, Vol. LX, No. 1, (1967), pp. 38-40.

Internal Organization

A student director, responsible for the coordination of the tournament, appoints chairmen to the various committees. The duties of these committees include the mailing of information to interested schools, registration of teams, grading of tests, and serving lunch.

One of the students' most difficult jobs is the construction of the tests as the quality of the problems plays a definite role in determining how well the tournament is received by the participants. Four months prior to the tournament, each committee member submits a specified number of problems for each of the ten categories and the team test. These are duplicated and each member must then evaluate each submitted problem rating it as to its difficulty and originality. Finally, the committee meets, and with a minimum of faculty supervision, selects five questions — three of lesser difficulty (to be worth three points each) and two of greater difficulty (worth five points each).

In each testing category it is tried to avoid duplicating the need for a single technique in problem solving. Consequently, events vary from year to year depending on the students' ability to collect creative problems. Certain topics have proved to be particularly restrictive and have had to be "retired" at the end of a few years. In addition, since most students follow a curriculum which allows the study of either calculus or analytic geometry, but not both, students choose between these two advanced topics. Thus for the calculus/analytics event, the students entered must select one and only one of the two different tests conducted simultaneously.

The Team

Each participating school is allowed to enter a team composed of ten members, including at least two sophomores and no more than four seniors. A maximum of four team members may enter each event and each member must enter between two and five of the ten events. By including a wide range of event topics, the sophomores are usually as necessary as the seniors. Because of these registration requirements, some ingenuity must be used in placing students in suitable categories. Each team, in planning its strategy, must fill out a registration form like the one shown here.

Team sponsors, if they wish, may enter into the testing in competition with other team sponsors. A sponsor chooses five of the ten events and takes the same test at the same time as the students are taking that test. In this contest no scores are announced, only an award made to the highest scoring sponsor.

The Day of the Tournament

A Bellaire student is assigned to each team as it arrives. This host helps register the team, sees that each of its members gets to the proper tests and, in general, assists the team in any way possible. After brief introductory instructions by the tournament coordinator, the testing begins.

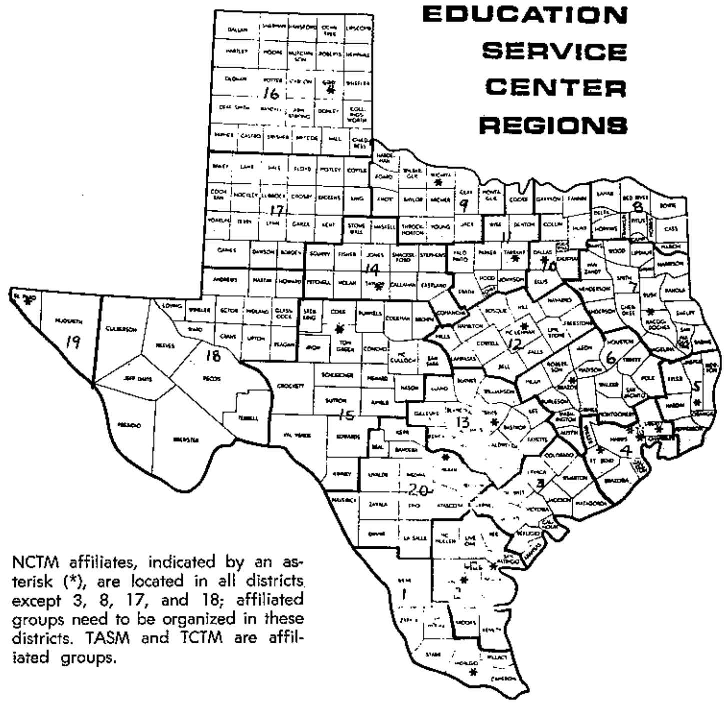
The students who are participating in an event proceed from the cafeteria to the testing room and are given test copies and specific directions. As soon as the examination begins extra test copies are taken back to the team members who are not entered in that event. In this way even those not being tested have a part in the excitement. At the conclusion of an event, the test papers are immediately taken to the grading room where each paper is graded twice to insure the greatest possible accuracy. Since the results are ready to be posted by the end of the next event the teams have an almost up to the event record of the results throughout the day. Only team scores are tabulated and team awards given. Because answers to each event are sent to the cafeteria along with the scores of each event, each school can check their score with the posted score.

The tests are each 15 minutes in length (not really enough time for the contestant to work all five problems in most cases) and the time between events is ten minutes. Except for a lunch break, events proceed one after the other. As scores are posted tension mounts for the final event – the team test. Each team is taken to a classroom and supplied with paper, pencils, and chalk. Instructions for the thirty minute test are given over the public address system. Since the test consists of several problems of different values, it often determines the final winner.

Evaluation

This tournament is designed to be a learning experience for both those students planning the contest and those participating in it. To Bellaire students the preparation phase represents a short course in business and management. The students who select the test problems obviously benefit by broadening their mathematical background. The participating students benefit through their practice sessions prior to the tournament. Indeed, these students normally study materials that are above their level of classroom instruction. Through the team awards, emphasis is placed on the total level of achievement and creativity in each school, rather than on the abilities of individuals.

	Grade	Beginning Algebra	Trigonometry	Radicals & Exp.	Triangles	Logarithms	Equations	Series & Prog.	Circles	Probability	Calc/Analytics
J. Brindley	Sr.		X			X	X			X	C
C. McGrane	Sr.		X	X			X				C
M. Tidwell	Sr.			X			X	X			A
T. Wheeler	Sr.		X	X			X			X	A
F. Carleton	Jr.		X			X		X	X		
C. Crowe	Jr.	X			X	X		X			
J. Furman	Jr.			X	X				X	X	
J. Handley	Jr.	X				X		X		X	
G. Busch	So.	X			X				X		
D. Goldberg	So.	X			X				X		



EDUCATION SERVICE CENTER REGIONS

NCTM affiliates, indicated by an asterisk (*), are located in all districts except 3, 8, 17, and 18; affiliated groups need to be organized in these districts. TASM and TCTM are affiliated groups.

Articles Wanted . . .

THE PRESIDENT'S MESSAGE

JAMES E. CARSON

I am sure that you had a nice summer vacation and are back in school by now. I had a nice trip to Europe.

It is press time again for another Journal. We sincerely hope that you liked the first one. How about comments? We want what you want and the only way that we can publish what you want is for you to let us know. We need articles for the forth-coming issues. Why not publish an article. From all over the entire state of Texas we must have many interesting things going on in our schools. We would like to hear from every section of the state.

As you can see, we have plenty of advertisers in the fall issue but we need a few more for the coming issues. If you know of anyone that would be interested in advertising in our Journal, please have him contact me for rates, deadlines, etc.

It is our sincere desire to have the Journal in your hands before the first T.C.T.M. workshop which is going to be held at J. Frank Dobie High School in the Pasadena Independent School District on October 23, 1971. We will accept registrations until October 18. Registrations before October 11 will be $5.00; after that date it will be $6.00. Send registration fees to Mr. Bill Ashworth, J. Frank Dobie High School, 11111 Beamer Road, Houston, Texas 77034. Come and make the first one a big success. We have engaged prominent people to conduct the workshop and we will have three sections: elementary, junior high, and senior high.

The C.A.S.M.T. meeting is not going to be held until December 9-11 so we need to concentrate on membership. You know that we usually make our big drive in the fall at C.A.S.M.T.; therefore, we are going to have to put forth more effort and get the membership earlier. I have sent each local organization some forms and if you need more, just mimeograph the front copy or use the one in your Journal. Again, may I stress the importance of membership. As of now, that is our only source of revenue.

I hope you have a great school year and I will see you at C.A.S.M.T.

PROFESSIONAL MEMBERSHIP APPLICATION

Date:________ School:________ School Address:________

Position: ☐ teacher, ☐ department head, ☐ supervisor, ☐ student,* ☐ other (specify)________

Level: ☐ elementary, ☐ junior high school, ☐ high school, ☐ junior college, ☐ college, ☐ other (specify)________

Other Information:________

		Amount Paid
State or Province ORGANIZATION:________	☐ New membership ☐ Renewal membership	2.00
Local ORGANIZATION:________	☐ New membership ☐ Renewal membership	
OTHER:________	☐ New membership ☐ Renewal membership	

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National Council of Teachers of Mathematics	Check one: ☐ New membership ☐ Renewal membership	↓
	$ 9.00, dues and one journal ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
	13.00, dues and both journals	
	4.50, student dues and one journal* ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
	6.50, student dues and both journals*	
	5.00 additional for subscription to *Journal for Research in Mathematics Education* (NCTM members only)	
	.50 additional for individual subscription to *Mathematics Student Journal* (NCTM members only)	
	The membership dues payment includes $4.00 for a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher* and 25¢ for a subscription to the *Newsletter*. Life membership and institutional subscription information available on request from the Washington office.	
* I certify that I have never taught professionally________ *(Student signature)*	*Enclose One Check for Total Amount Due* →	

Fill out, and mail to Dr. Floyd Vest, Mathematics Department, North Texas State University, Denton, Texas 76203. Make checks payable to the Texas Council of Teachers of Mathematics.

NOW!!

NOMINATING COMMITTEE

The nominating committee is composed of:

Mrs. Pat Echols of Spring Branch

Mr. Radford Gregg of Fort Worth

Mr. Bill Ashworth of Pasadena

They have made the following nominations for the slate of officers for the coming year:

President-Elect: Mrs. Shirley Ray, Corpus Christi.

Vice-President: Mr. Crawford Johnson, Jr. High teacher, Fort Worth

A UNIFORM METHOD FOR FACTORING THE GENERAL TRINOMIAL

by HARLAN SMITH

Polynomials of the form $rx^2 + sx = t$ where r, s, and t are integers and $r = 1$, have traditionally been factored by reversing the steps followed in finding the product of two binomials. That is, students would find factors of t whose sum was s.

The distributive property is used in multiplying two binomials, and it is the intent of this article to show that using the distributive property and a knowledge of simple factorizations leads to a single uniform method for factoring general trinomials with integral coefficents. So called "trial and error" methods are unnecessary and solutions are quickly derived from a factorization of integers.

Let us observe the general form of the product of two binomials.

$$(ax + b)\ (cx + d) = acx^2 + adx + bcx \quad + bd$$
$$= acx^2 + aad + bc)x + bd$$

Comparing this to the polynomial form $rx^2 + sx + t$, we see that $r = ac$, $s = ad + bc$, and $t = bd$. When $r = 1$, note that the product of $rt = t$. In that case, students proceeded to find factors of t whose sum was s. Will this procedure hold when r has integral values other than one? Substituting for r and t and applying commutative and associative axioms, we have $r \times t = ac \times bd$ and $rt = ad \times bc$.

Thus, the factors of rt (ad and bc) do form the addends for s ($ad + bc$). Thus, to factor any trinomial over the integers, one needs to find factors of rt whose sum is s. If such factors do not exist, then the trinomial is prime.

Example 1: Factor $x^2 - 8x + 15$.

$r = 1$, $s = -8$, $t = 15$, and $rt = 15$.

Factors of 15 are: $1 \times 15, -1 \times -15$
$3 \times 5, -3 \times -5$

The last factorization has the desired product and sum, and we write:

$$x^2 - 3x - 5x + 15$$
$$= x(x-3) - 5(x-3)$$
$$= (x-5)\ (x-3)$$

Example 2: Factor $12x^2 + 79x - 35$

$r = 12$, $s = 79$, $t = -35$, and $rt = -420$

Find factors of -420 whose sum is 79.

Factors of -420 are:

$1 \times -420, -1 \times 420$
$2 \times -210, -2 \times 210$
$3 \times -140, -3 \times 140$
$4 \times -105, -4 \times 105$
$5 \times -84, -5 \times 84$

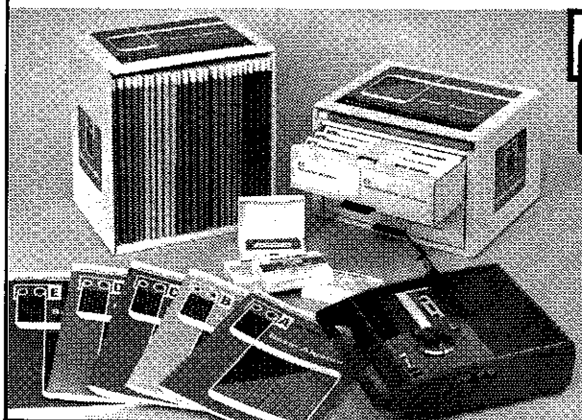
Write:

$$12x^2 - 5x + 84x - 35$$
$$= x(12x-5) + 7(12x-5)$$
$$= (x+7)\ (12x-5)$$

(Continued on Page 19)

New Cassette Teaching Tape Programs from Charles E. Merrill

1 Cassette Tape + 1 Student Study Booklet + 1 Cassette Player = 1 Self-Study Classroom



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This modern cassette teaching tape program provides for independent study of topics most frequently included in a first-year algebra course. The eighteen cassette tapes and five student books complement each other so that the student both sees and hears the mathematics to be learned. This simultaneous sight and sound presentation has proved especially helpful for students whose mathematical difficulties are related to a lack of essential reading skills.

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By Francis T. Sganga

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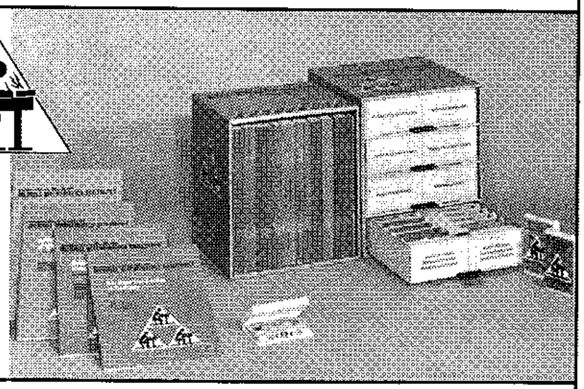

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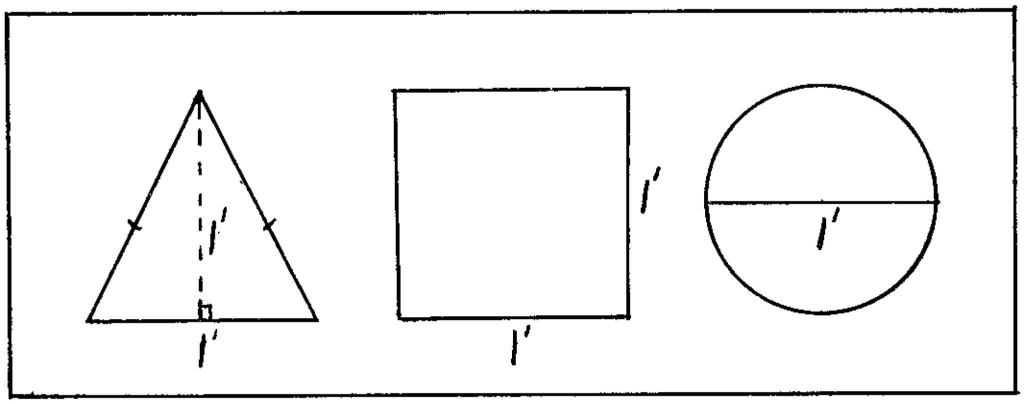
THREE DIMENSIONAL FIGURE PUZZLE

Design a **single** 3 dimensional figure that will exactly fill and pass through all three holes in the screen below:



Example: A cone with d_{base} = 1′ and h = 1′ could fill the circle one way and the triangle another way, but would not fill the square.

RESULTS OF TASM MARCH 29 SURVEY

1. The revised TEA bulletin should [18] suggest [5] designate [2] not suggest a sequence of mathematical courses.

2. If a "suggested sequence of mathematics courses" is desired, which of the following sequence is preferred?

   [16] Algebra I → Geometry → Algebra II

   [6] Algebra I → Algebra II → Geometry

   [2] Both

3. Do you recommend Algebra I be taught in an accelerated 8th grade program?

   [17] Yes [8] No

4. If your response to question #3 is yes, which of the following courses should follow 8th grade Algebra?

   [13] Geometry [4] Algebra II

5. Do you favor?

   [14] All courses being awarded credit on ½ unit basis (Algebra 1, 2, 3, 4 would replace the present Algebra I, II)

   [10] Some courses being awarded ½ unit basis

   [1] Other <u>Only Algebra 3, 4 on ½ unit basis</u>

   [1] <u>Only if review of first half in second half</u>

6. Consumer Mathematics should be:

   [12] A course for the terminal student to meet minimum graduation requirements

   [7] An elective course for 11th and 12th grade students who have completed Algebra I

   [5] Both

   [1] Elective but not Algebra as prerequisite

7. Should Related Mathematics I (Algebraic) and Related Mathematics II (Algebraic) be equivalent to Algebra I?

   [20] Yes [4] No [1] Eliminate entirely

8. Minimum graduation requirements in Mathematics should require:

   [18] Any two courses

   [6] Related Mathematics II, Geometry or Consumer Mathematics

   [1] Other: Three credits

RESULTS OF TASM MARCH 29 SURVEY (Continued)

9. What type of 10th grade course do you recommend as a successor to Related Mathematics I in which Gold's *Modern Applied Mathematics* is the textbook?

   11 – Consumer Mathematics

   8 – More Applied Math

   Elementary probability and statistics

   Elementary number theory

   Home Economics Math

   Consumer, vocational and domestic uses of mathematics, number theory, business law and elementary statistics on a basic level.

   A course which integrates science and technology using a laboratory approach including:

   a. Extend geometry to include a more extensive treatment of ratio and proportion, all of the geometric constructions and arguments to support their validity.

   b. Extend systems of measurement to include "derived" systems; pressure, electrical units, etc.

   c. Introduce concepts from probability and statistics to get at sampling theory and normal distribution

   d. Introduce simple systems of linear equations and a discussion of linear programming

   e. Discuss and use various mechanical and electronic devices for computation.

10. The writing committee will include independent study courses such as linear algebra, linear programming, and number theory which may be taken by students on a seminar basis. What additional courses should be considered?

8 – Probability	Math Modeling Techniques
8 – Statistics	Advanced Arithmetic (Refresher Mathematics)
4 – Matrix Algebra	Algebra of Sets
2 – Calculus	Polynomial Functions
2 – Topology	College Algebra
2 – Vectors	Advanced Geometry
2 – Polynomial Functions	Non Euclidean Geometry
2 – Boolean Algebra	Projective Geometry
Finite Math	Solid Geometry
Math – A Human Endeavor	Slide Rule
Technical Math	Game Theory
History of Math	Groups and Their Graphs
Celestial Mechanics	

   Trig. ½, Analytical Geometry ½, Elementary Analysis ½ or 1, Calculus ½ or 1

   Each school should take care of this.

   Little or no need for individual study programs.

   Any course a student wishes to take or in which there are not enough to make a course.

SOME SPECIAL TECHNIQUES IN WORKING WITH SEQUENCES

W. D. CLARK

One of the most valuable assets a person may have is the talent of pattern recognition and characterization. This talent is brought to bear in most human endeavors. Weather patterns, cultural patterns, hereditary patterns, writing styles, artistic styles, and voting patterns are but a few examples of patterns sought by man in order that he may make predictions based on his findings.

In mathematics, a natural vehicle which allows one to exercise and develop this aforementioned talent is the study of sequences. Moreover, through sences have been chosen, which it is hoped will be of limit and the study of convergence.

For the purpose of this paper we will consider a sequence to be defined when a listing of numbers is given in sufficient length so as to recognize a definite pattern. In practice this pattern recognition is ascertained by one's ability to write the "next" term.

For this article two topics in the study of sequences have been chosen, which is is hoped will be of interest to mathematics educators.

I. TWO TECHNIQUES FOR WRITING THE GENERAL TERM OF A SEQUENCE

In many cases even though a pattern is easily recognizable, the problem of writing a formula for the general (or n^{th}) term is quite formidable. In some cases this difficulty is overcome by simple techniques which are overlooked by many students merely because they are subtle techniques and perhaps have never been explicitly pointed out to these students. As an example we shall consider two sequences and trust that the reader will be able to apply this technique to appropriate sequences.

Sequence:

$$1, 2-1, \frac{1}{2}, 2-\frac{1}{2}, \frac{1}{3}, 2-\frac{1}{3}, \ldots \qquad (1)$$

Sequence:

$$1, 1-\frac{1}{2}, 1+\frac{1}{2}, \frac{1}{2}, 1 \quad \frac{1}{3}, 1+\frac{1}{3}, \frac{1}{3}, \ldots \qquad (2)$$

In (1) we see an overall or parent pattern and two sub-patterns; one in the even terms and one in the odd terms. Similarly, in (2) we see a parent pattern and three sub-patterns. Sequences of this type may be characterized most easily by writing formulas for several "general" terms rather than one.

Returning to (1) we see that if we can characterize the pattern in the even terms and the pattern in the odd terms then we will have characterized the parent sequence. If a_n denotes a generic term of the sequence then any generic even term will be of the form a_{2k} and any generic odd term will be of the form a_{2k-1}. Consequently these terms may be characterized by the formulas;

$$a_{2k} = 2 - \frac{1}{k} \text{ and } a_{2k-1} = \frac{1}{k} \quad k = 1, 2, \ldots .$$

These two formulas then characterize the parent sequence. Note that the characterizations started with the 2k terms; the terms just prior to the parent pattern repetition.

Looking now at (2) we see that the 3k terms are the terms just prior to the parent pattern repetition. Following the procedure above, this sequence may be characterized by the formulas;

$$a_{3k} = 1 + \frac{1}{k+1} \text{ and } a_{3k-1} = 1 - \frac{1}{k+1} \text{ and}$$

$$a_{3k-2}\ 2 = \frac{1}{k} \quad k = 1, 2, \ldots .$$

The key to this technique is characterizing the term just prior to the parent pattern repetition and then working backwards. This is generally easier than trying to write a formula for a general term immediately.

Another technique which is sometimes useful and which may also be used as enrichment material is a technique which relies on methods and results of difference calculus. Consider the sequences;

Sequence: 4, 8, 12, 16 . . . (3)

Sequence: 1, 3, 6, 10, 15, 21, . . . (4)

One may begin his characterization in several different manners but we have chosen to characterize these sequences in a way that will exemplify a "difference equation" technique.

Beginning with (3) we may characterize the sequence recursively by the formulas; $a_{n+1} = a_n + 4$ or $a_{n+1} - a_n = 4$ or, by using the notations of difference calculus, by $\Delta a_n = 4$. (5)

The difference operator in difference calculus is analogous to the differential operator of calculus and the solution of a difference equation is somewhat analogous to the solution of the corresponding differential equation. The solution to (5) is:

$$a_n = 4n + c$$

and in order for this to characterize (3) we see that c must be zero. Also, the formula $a_n = 4n$ is a more concise way of characterizing (3) than the recursive formulas.

Looking now at (4) and using the method above we have;

$$a_{n-1} = a_n + (n+1) \text{ or } \Delta a_n = n + 1. \quad (6)$$

The solution to (6) is $a_n = \dfrac{n(n+1)}{2} + c$

where again we see that c must be zero. The corresponding differential equation is $f'(x) = x + 1$ whose solution is

$$f(x) = \frac{(x+1)^2}{2} + c \quad = \frac{(x+1)\ (x+1)}{2} + c.$$

I have found that when this technique is shown in a classroom, students are eager to know more about difference equations and are quite willing to study difference calculus on their own.

II. A SPECIAL THEOREM

One theorem which is particularly useful when sequences are characterized by more than one formula is a theorem dealing with subsequences -of-a-sequence and limits. Before the theorem is stated and exploited some examples are given which will clarify the terminology used.

If a sequence is determined by a listing of numbers then by a subsequence of this parent sequence is meant, rather intuitively, a second listing of numbers such that each term of the second listing is a term of the parent sequence and that the terms of the second listing are arranged in the same order in which they appear in the parent sequence. As an example;

Parent Sequence: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots.$

A Subsequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots.$

Another Subsequence: $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots.$

Another Subsequence: $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \ldots.$

Another idea for which an intuitive explanation is given is the concept of a sequence being "broken down" into subsequences. By a sequence being "broken down" into a finite number of subsequences we mean that there exists (and in fact in many, many ways) a finite number of subsequences having the properties that;

Each term of the parent sequence appears once once and only once in the total listings of the subsequences.

To illustrate this concept the sequence;

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.$$

may be "broken down" into the two subsequences

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots. \qquad \text{and}$$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots.$$

The theorem referred to earlier may be stated in the following manner: If a sequence may be "broken down" into a finite number of subsequences then all of these subsequences converge to the same limit, L, if and only if the parent sequence converges to L.

Returning now to the two earlier sequences (1) and (2) we see the sequence

$$1, 2-1, \frac{1}{2}, 2-\frac{1}{2}, \frac{1}{3}, 2-\frac{1}{3}, \ldots. \qquad \text{may be}$$

"broken down" into the two subsequences

$$2-1, 2-\frac{1}{2}, 2-\frac{1}{3}, \ldots. \qquad \text{and}$$

$1, \frac{1}{2}, \frac{1}{3}, \ldots.$ which were characterized by the formulas;

$$a_{2k} = 2 - \frac{1}{k} \qquad \text{and} \qquad a_{2k-1} = \frac{1}{k}.$$

(Continued on Page 19)

DANGEROUS JOURNEY

... An Adventure Into the Land of Numbers

CAROLYN H. ADAMS

We went on a journey today, my class and I; a dangerous journey. Our paths curved through the Black Forest, past the moat of the evil witch and precariously skirted quicksand and the bottomless pit. We took along with us our four mathematical friends, A,S,M and D, finding them effective tools to aid us on our journey.

What fun it was to begin with bankrolls of our own choosing and vie our arithmetic skills against others along the way. When at times we felt secure in our standing, and were faced with five stacks of face down cards, it was most tempting to choose the one with the question mark, for we and we alone were masters of our fate (or so we thought!) along the pathway.

Our "playing board," a three by four foot framed piece of pegboard, contained sufficient holes to allow our golf tee "players" to advance down their separate paths. And it was large enough for the whole class to follow and play along.

Interested? Then come join us on our dangerous journey into the land of numbers. Everyone can play; and all at the same time. Let's choose four of us, however, to actually make the moves. These will have to make all of the decisions while the rest of us keep an account of what is going on. We can only hope to learn from their mistakes so that when it is our turn to take the journey, we can do a little better!

Now, the four players are seated in front of the class at a small desk on which are five stacks of cards. Four of these stacks each contain eleven cards numbered zero through ten. On the back of each set is either an ×, +, —, or ÷, indicating what is to be done when these are drawn. While the fifth stack has only a question mark on the back of each card. The player may be told to do almost anything with these cards, such as "go straight to the bottomless pit," or "divide your score in half" or whatever the students might wish to add, since it may contain any number of cards.

The teacher asks each player, now, which path he would choose for the journey; each path showing place value in its placement right to left (ones, tens or hundreds). Then she asks what three bills they choose; a one dollar, ten dollar or hundred dollar bill or any combination, as long as they "receive" only three bills. One player may decide on three ones ($3.00), another, perhaps, two tens and one one ($21.00) and still another may wish one of each kind ($111.00). When they decide, they must state the sum. Then the teacher has the score sheet set up on the board such as:

Jack: red tens path $110.00	Mike: gold tens path $300.00
Betty: yellow ones path $21.00	Frank: blue ones path $120.00

This, then, is copied by the players as well as the rest of the students, or Board of Experts, to be used as their tally sheet.

A student is chosen as "charter" to move the tees on the game board and read out the instructions on each jump; and another student is assigned the number of moves each player is to take. This is made simply with a square piece of cardboard marked with the numerals one to ten and a moving bit of plastic or metal braded to the center of the circle.

A player adds to or subtracts from his score as he advances on the path he has chosen; on the ones path by ones, on the tens by tens and on the hundreds by hundreds. If he lands on a square marked DT, it means draw and tally. He must choose from one of the stacks on the table before him. If his total has gone down due to landing on minus squares he might wish to draw from the addition stack, thus adding from zero to ten dollars to his score, or if he is terribly below his original score, he might wish to draw from the multiplication stack, multiplying whatever he draws by the number the spinner shows to be his move and adding this to his score. A number from the division pile divides his total score and one from the subtraction pile merely subtracts whatever numeral it indicates.

Whenever a player lands on a square marked

TC, this is for toll check and means he must state his present total. The Board of Experts can then agree or disagree, giving us an accurate check of the computation to that point. If the Board agrees the score given is incorrect, the player automatically goes to either the bottomless pit, quicksand, black forest, or moat of the evil witch; whichever is the closest. There he awaits his next turn. After his spin has been made and told him, he may indicate whether he wishes to be placed back on his previous place on the board by multiplying five times his current spin and adding this to his score. This he may wish to do unless he needs more added to his score, or wishes to gamble that his next spin may be lower. The next spin (his next turn), however, is to be multiplied by ten and added to his score. This he must accept and then be placed back on the board.

At various places along the way are squares to send players to the pit, moat, quicksand and forest, so it is at all times a dangerous journey!

Halfway through the journey is a toll check house where players wait for the others to catch up and thus begin their second half of the journey together. As they land on this, they give their present score and are checked by the Board of Experts. All of these toll checks keep everyone on their toes and the game more exciting. We all like to feel a part of what is going on and with the ups and downs of our players and their decisions which meet them at every turn, it is an exciting day indeed when we all go on a dangerous journey.

The winner is determined by the player whose sum is closest to his original sum when all have reached "the hills of home." With a fourth or fifth grade group, I encourage them to use the ones or tens path as they make the journey, but an older group more familiar with negative as well as positive numbers would be well challenged with the hundred's path.

Any way you travel, however, it is hoped you enjoy yourself while you develop your arithmetic skills, and that you will take time another day to try this trip into the magical land of numbers.

☆ SPECIAL TECHNIQUES WITH SEQUENCES (Continued from Page 17)

By considering these formulas it is readily seen that the first subsequence converges to 2 and the second to 0. Hence, exploiting the theorem, it is seen that the parent sequence does not converge.

Similarly, the sequence

$$1, 1-\frac{1}{2}, 1+\frac{1}{2}, \frac{1}{2}, 1-\frac{1}{3}, 1+\frac{1}{3}, \frac{1}{3}, \ldots$$

may be "broken down" into the three subsequences

$$1+\frac{1}{2}, 1+\frac{1}{3}, \ldots \quad \text{and}$$

$$1-\frac{1}{2}, 1-\frac{1}{3}, \ldots \quad \text{and}$$

$$1, \frac{1}{2}, \frac{1}{3}, \ldots$$ which were characterized by the formulas;

$$a_{3k} = 1 + \frac{1}{k+1}, \; a_{3k-1} = 1 + \frac{1}{k+1}$$

$$\text{and } a_{3k-2} = \frac{1}{k}.$$

Once again, by considering these formulas it follows that the parent sequence does not converge.

In sequences of this type it is far easier to consider the limits of the subsequences than that of the parent sequence and thus points out to the student the actual advantage of characterizing the sequence by several formulas rather than one. I welcome this point in the study of sequences since students generally have a feeling that it is somehow "dishonest" to characterize a sequence by more than one formula.

Dr. W. D. Clark, Associate Professor
Stephen F. Austin State University
Department of Mathematics
SFA Station, Box 3040, Nacogdoches, Texas 75961

☆ GENERAL TRINOMIAL (Continued from Page 9)

Or alternately:

$$12x^2 + 84x - 5x - 35$$
$$= 12x\,(x+7) - 5\,(x+7)$$
$$= (12x-5)\,(x+7)$$

This method is uniform, simple to follow, eliminates lengthy "trial and error" combinations, and reveals prime trinomials at an early stage.

Submitted by: Harlan Smith
Consultant for Mathematics
Lubbock Public Schools
1618 19th Street
Lubbock, Texas 79401

TEXAS MATHEMATICS TEACHER
J. William Brown, Editor
Texas Council of
Teachers Of Mathematics
Woodrow Wilson High School
100 S. Glasgow Drive
DALLAS, TEXAS 75214

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